Using Brief Experimental Analysis to Identify Effective Mathematics Fluency Intervention for Students in Middle School

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Dedication

This dissertation is dedicated to my eldest brother MouaYing Mouanoutoua, who taught me the value of education. It is because of him that I have come to understand how important it is to live life to the fullest and to take the road less traveled which ultimately led me to my educational success. This dissertation is also dedicated to my wife Ia Ong Mouanoutoua, who resembles my mother in many ways; I am especially grateful for a loving and supportive wife.

ABSTRACT

Recently, scholars have utilized brief experimental analysis (BEA) as a way to efficiently identify math interventions that are effective in supporting elementary students' performance in mathematics. Although BEA research in mathematics performance of students in the elementary grades is emerging, far less application has been done with older adolescents. Considering the limited number of math studies involving the use of BEA for older students, the purpose of study evaluated the use of BEA to identify effective multiplication fluency intervention for students in middle school. Four participants in middle school served as participants throughout this study. The effects of multiple math interventions on single digit multiplication facts were assessed for each participant using a BEA. For each participant, the most effective intervention identified by the BEA was further evaluated overtime using a multiple baseline probe during an extended analysis. Visual analysis of the data collected during the BEA and the extended analysis suggest that the BEA identified intervention supported participants' acquisition of their targeted facts but performance on maintenance and generalization to inverse and division problems was mixed. Results from this study provide support for the use of BEA to efficiently identify effective multiplication fact interventions for individual students.

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Chapter 1

Introduction

In mathematics, procedural fluency is one of the five interdependent strands (see Figure 1) that must be acquired when becoming math proficient (Kilpatrick, Swafford, & Findell, 2001). The other four strands are: conceptual understanding, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). Procedural fluency is defined as the ability to apply procedures or strategies accurately and efficiently; to generalize procedures to novel problems and contexts; and to recognize when one strategy or procedure is more effective than another (National Council of Teachers of Mathematics [NCTM], 2014; Common Core State Standards [CCSS]). Put another way, procedural fluency of basic math facts involves carrying out procedures according to the following four tenets: (1) flexibly, (2) accurately, (3) efficiently, and (4) appropriately. In order to assess basic fact fluency, all four tenets of procedural fluency must be addressed (Kling & Bay-Williams, 2014). Examples of procedural fluency tasks include identifying the most appropriate strategy in deriving the following facts (e.g.7 + 8 = ___ or 8 x 6 = ___) within a certain amount of time.

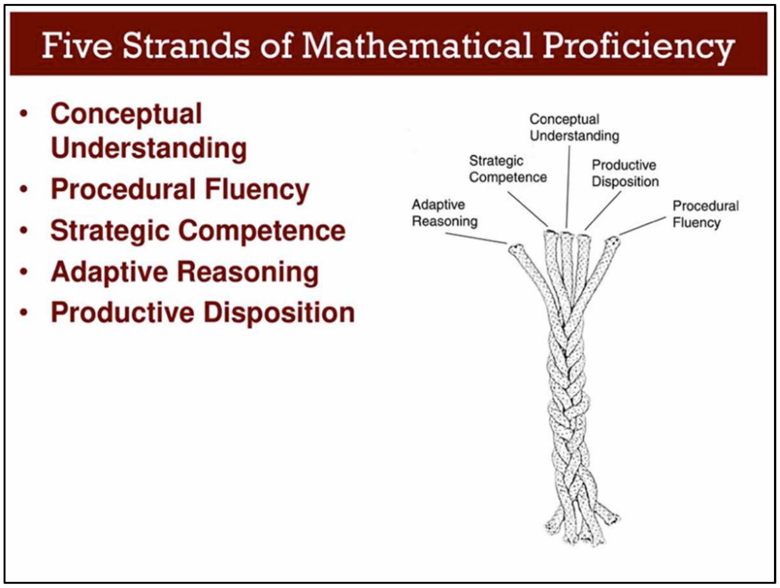

Figure1. An illustration of Kilpatrick, Swafford, and Findell's (2001) Five Strands of Mathematical Proficiency. Retrieved from https://slideplayer.com/slide/12788242/

According to the Common Core State Standards for Mathematics (2010), procedural fluency for basic math facts (e.g. multiplication) should be mastered in the early primary years. However, students across the country continue to experience difficulty acquiring basic math facts on a daily basis and have heavily depended on calculators for basic computation (National Mathematics Advisory Panel [NMAP], 2008; Leach, 2016). Additionally, it has been reported that students in the United States perform significantly lower on math tests than students from other industrialized nations (Jitendara, Salmento, & Haydt, 1999). Part of the underlying issue is that students lack the *acquisition* and *fluency* of early skills that impede their efforts toward subsequent procedural fluency in basic computations. Importantly, this lack of skills begins in elementary school and often persists into middle and high school (Rivera & Bryant, 1992; Woodward, 2006; NMAP, 2008). Students' lack of basic math fact acquisition and fluency is reflected in data suggesting only 54% of 14-year olds in secondary schools have mastered multiplication fact fluency (Cawley, Palmer, Yan, & Miller, 1998; Fuchs & Fuchs, 2001). Acquisition, in this case, refers to the accuracy which determines if a student “can do” a problem correctly every time. Fluency, not to be confused with procedural fluency, is the ability of consistently stating a fact correctly within 2-3 seconds (Burns, 2005; Burns et al., 2015). Therefore, acquisition and fluency mirror the tenets of accuracy and efficiency within procedural fluency as defined by the CCSS (2010) and NCTM (2014) (see Table 1).

Table 1

Equivalencies of procedural fluency elements between the National Council of Teachers of Mathematics and the Common Core State Standards

NCTM	CCSS
Apply procedures or strategies accurately and efficiently	Efficiency and accuracy
Generalize procedures to novel problems/context	Flexibility
Recognizing when one strategy or procedure is more effective	Appropriate strategy use

Problem Statement

Students who are not yet fluent in basic math facts are likely to struggle with more advanced mathematical computations that require critical thinking such as solving word problems in later years (Woodward, 2006). Furthermore, when considering the relatively low math performance of students in the United States, teachers are encouraged to use effective evidence-based interventions that have been proven to support students who struggle with basic math skills (Fuchs & Fuchs, 2001). Unfortunately, identifying and implementing an evidence-based intervention that meets the needs of individual students can be a daunting and time-consuming task for practitioners.

It has been reported that teachers may spend valuable instructional minutes, up to one-third of classroom time, providing math remediation which may or may not be effective in meeting individual student needs (Carpenter, 1985). Furthermore, it has been reported that remedial instruction may be ineffective for students who have struggled with math for an extensive length of time because of a teacher's repeated failed attempts to implement effective instruction or evidence-based interventions (Chard et al., 2008). To combat the valuable time lost and help teachers become more effective, practitioners need a method to quickly identify an effective individualized intervention.

A prescriptive and research-based method for testing relative effects of evidence-based interventions on an individual's academic performance with the goal of identifying the most effective and efficient individualized intervention is *brief experimental analysis* (BEA; Coolong & Wagner, 2015). The bulk of the literature supporting the use of BEA to identify effective academic interventions has primary focused on testing reading interventions such as repeated reading, goal setting, incremental rehearsal, reward, and listen passage preview to improve reading fluency (e.g. Daly et al., 1998; Bonfiglio et al., 2004; McComas et al., 2009). In contrast, the literature base for using BEA to identify math interventions is limited considering how effective is has been for identifying reading interventions for individual students. In addition, BEA research to identify math interventions for middle school students who struggle with basic multiplication facts remains scant.

Purpose of the Study

Few BEA math studies exist and of those that are available, the majority of study participants have been younger elementary students (e.g. Codding et al., 2009; Mong & Mong, 2012). Because very few math studies involve older students (Hughes, Maccini, & Gagnon, 2003), the purpose of this study is to examine the use of BEAs to identify an effective intervention to teach multiplication facts to middle school students (i.e. grades 6th, 7th, 8th) who have not yet acquired proficiency with basic multiplication facts. A second purpose is to identify the extent to which BEA-identified interventions produce gains that generalize to novel problems and maintain over time.

Research Questions

This investigation was designed to answer the following questions:

(1) What is the effect of a math intervention identified during a brief experimental analysis (BEA) on multiplication fact fluency for middle school students who are performing below expectations with regard to basic multiplication facts

(2) What is the effect of the BEA-identified intervention on generalization to inverse multiplication facts and single-digit division problems, and

(3) To what extent do gains from the BEA-identified intervention maintain over time?

Chapter II

Review of the Literature

Instructional Hierarchy

When students learn an academic skill, they usually experience mastery of that particular skill through a progression of learning stages. Haring, Lovitt, Eaton, and Hansen (1978) named these learning stages the "instructional hierarchy" (IH). The IH consists of four stages, which include: (a) acquisition, (b) fluency, (c) generalization, and (d) adaptation. For any skill, a student may be at one of the four stages of this hierarchy.

In the first stage, acquisition, the student learns the fundamentals of the target skill. The focus of the acquisition stage is to establish or increase accuracy, which can be accomplished through providing teacher modeling, prompting, and teacher feedback with praise (Haring et al., 1978; Daly et al., 1996). Before mastering the second stage, fluency, the student is able to complete the targeted skill with high accuracy, but their performance of the skill is not yet fluent. Imagine a piano player who can read and play each note of a composition but does not string them together to play the piece fluently. Therefore, the focus of the fluency stage is to increase smooth and seemingly 'automatic' accurate performance, which can be accomplished by providing the student with interventions that often involve repetition (e.g., repeated reading, incremental rehearsal). In the third stage, generalization, the student has acquired and can perform the skill fluently but is not yet using the targeted skill within a different context or setting. Therefore, the focus of the generalization stage is to increase the generalizability of the acquired targeted skill by introducing interventions such as schema-based instruction

(SBI) to support the student's use of the skill throughout a wide range of context or settings. This particular stage can be associated with flexibility, a tenet of math procedural fluency. The final stage, adaptation, is where the student should be able to adapt the newly mastered skill and synthesize it with other skills to solve novel complex problems. Students who struggle in this stage can be provided with support to synthesize strategies and combinations of strategies to solve novel complex problems. This particular stage can be associated with appropriate strategy use, a tenet of math procedural fluency. These four stages of the IH outline a student's academic skill development and when an instructor identifies that a student is in a particular stage and struggling to be successful, specific types of instructional interventions can be selected to address the student's needs in that stage (Haring et al., 1978).

Instructional Hierarchy for Math Acquisition and Fluency

In mathematics, the IH can guide teachers to match intervention supports to a student's specific instructional needs. For example, if a student is working on basic math facts like "6 x 5" the IH calls for the student to first work on acquiring this fact through teacher modeling and feedback. A teacher can model solving this fact by using the factors of 5 strategy or teaching the student that 6 multiplied by 6 is 36 therefore 6 multiplied by 5 is 6 less than 36, or 30. Interventions that can support the acquisition of this skill may include Math to Mastery ([MTM] Mong and Mong, 2010; 2012).

When students can solve math facts accurately, they can then move to stage two of the IH, which calls for fluency development. Providing an abundant number of opportunities for practice and measuring digits correct per minute ([DCPM];

VanDerheyden & Burns, 2009) can support fluency development because speed and accuracy with which skills are completed (i.e. accuracy plus speed) are elements of fluency (Binder, 1996). Explicit timing and taped problems are interventions that have been shown to assist students in developing fluency with basic math facts.

Researchers have evaluated a skill-by-treatment paradigm using the IH to justify the use of certain math interventions when addressing students' specific needs. In a meta-analysis of acquisition and fluency math intervention publications, Burns, Codding, Boice, and Lukito (2010) investigated the evidence for a skill-by-treatment interaction paradigm. The authors examined the effectiveness of acquisition and fluency-based math interventions across 17 total studies. They compared treatment effects to baseline performance on math tasks such as single digit multiplication facts and had determined that third grade students who score 14 or fewer correct digits per minute are considered to be in the frustration level whereas 15 or more correct digits per minute constitutes an instructional level. Likewise, fourth grade students who score 24 or fewer digits correct per minute are considered to be in the frustration level and 25 or more are considered to be in the instructional level. Part of the author's purpose was to determine whether initial assessment of skill level could be useful for instructional planning as suggested by Martens and Eckert (2007). Results from their meta-analysis suggest that students who are performing at the frustration level are better suited with acquisition interventions whereas students who are performing at the instructional level should be provided with fluency interventions. Results from this meta-analysis are consistent with Codding et al., (2007) who demonstrated that acquisition interventions such as cover-copy-compare were more effective for students who are still trying to acquire the skillset (i.e. computing math facts) but that fluency interventions such as explicit timing are better suited for students who have already learned the computational skill and are in the instructional level range but have not achieved computational fluency. Finding from Burns et al., (2010) support the notion that in math, the stages of the IH in which the student is functioning can inform the type of intervention to be used (Rivera & Bryant, 1992).

The IH can be used to guide matching interventions to student needs in terms of basic math skill acquisition and fluency (Burns et al., 2010) but more research is needed to fully comprehend how the IH can assist in matching interventions to student needs in terms of developing conceptual knowledge, procedural fluency, and automaticity in mathematics. As an alternative, researchers have used a heuristic approach to explore the effects of matching interventions to students' mathematical needs in regard to conceptual and procedural understanding. For example, Burns et al. (2015) examined the use of a conceptual and procedural framework to support the identification and implementation of math interventions for three elementary students who had math deficits. Two first grade students were working on addition facts and the third-grade student was targeting single digit multiplication facts. Students were assessed for conceptual understanding of underlying concepts by comparing visual and algebraic representations of the same problem to identify which numerical equation represented the visual representation of the problem. For example, can students match a picture of a die with 6 dots plus a die with 3 dots to a written equation of 6 + 3 = ___? If they met an established criterion (i.e. 90%), they were provided with a procedural based intervention (i.e. incremental rehearsal).

Students who did not meet criterion were provided with a conceptual based intervention (i.e., Build in Parts, Fill the Chutes). The authors measured digits correct per minute and results of the prescribed intervention showed immediate performance gains over baseline. The authors employed a contra-indicated design to measure and compare the effects of the non-prescribed intervention. To illustrate, if students were prescribed a procedural-based intervention because they met criterion then they were also provided a conceptual intervention to examine which intervention produced the most impact. Results indicated that the prescribed intervention produced more student growth when compared to the non-prescribed interventions. The study conducted by Burns et al., (2015) discussed that students who struggle with math procedurally should be provided with procedural based interventions and students who struggle with math concepts should be provided with conceptually based interventions. This notion builds upon the skill by treatment paradigm that suggests instructional support should match a students' frustration or instructional performance levels in mathematics.

Acquisition and Fluency-Based Mathematics Intervention

One important component of mathematical competence is fluency with basic operations (Baroody, 2006). Elementary students must acquire fluency with basic math skills (i.e., addition, subtraction, multiplication, and division) in order to solve more complex problems (National Council of Teachers of Mathematics [NCTM], 2000). When students experience deficits in basic math skills, learning gaps increase as students transition from elementary to middle and high school. Students who lack fluency with basic math skills in early grades continue to struggle in later grades as teachers emphasize higher-order operations (Council of Chief State School Officers & National Governors Association [CCSSO & NGA], 2010). To counteract this deficit, support should be provided to enhance students' mastery of basic math facts so that students can minimize cognitive load (i.e., working memory) when presented with higher-level mathematical problems.

Neurobiological evidence points to the importance of mastering basic math facts because mastery of math facts can reduce cognitive load when deriving novel problems. Studies have shown that during single digit computation, students with higher math achievement experienced activation in areas of the brain related to recall rather than processing (Price, Mazzocco, & Ansari, 2013). In other words, students who can automatically recall math facts can efficiently and effectively channel cognitive load towards solving higher level math problems whereas students who cannot recall math facts will have a more difficult time balancing cognitive load by first using mental capacity to compute math facts prior to solving mathematical problems.

Within an RtI framework, when older students lack fluency with basic math skills, screening and progress monitoring combined with an understanding of where in the IH a student is performing a particular skill can provide guidance pertaining to specifying an individualized intervention. When educators encounter students who failed to master basic skills in earlier grades, they must utilize assessment procedures that will quickly identify an effective approach to instruction to accelerate the student's acquisition, fluency, and generalization of the basic skill. In turn, when students have been appropriately supported, they can become more successful in adapting and applying basic math skills to more complex mathematics, such as algebra, during later years. Multiple interventions exist that have been demonstrated to have at least a short-term positive effect on improving a student's accuracy and fluency skill (Carson & Eckert, 2003). The following are examples of interventions designed to improve acquisition (e.g. accuracy) and fluency (e.g. efficiency) of basic math facts.

Math to Mastery

Math to Mastery (MTM) is an intervention that targets math computational fluency (Mong & Mong, 2010). MTM incorporates a number of intervention strategies that are research based which include (a) previewing the problem, (b) repeated practice, (c) corrective feedback, (d) performance feedback, and (e) progress monitoring (Mong, & Mong, 2012). Strategies of the MTM intervention can best be implemented by first providing the interventionist and student the same problem worksheet. Within a session, the interventionist models how to compute the problem while the student observes (i.e., previewing the problem) making sure to explicitly state each step in the procedure. Next, the student practices completing the same problems more than once (i.e., repeated practice). As the student works out the problem, the interventionist observes and provides correction as needed (i.e., corrective feedback). After the student successfully completes all the problems within a session, the interventionist compares and provides updates of the student's performance to previous sessions (i.e., performance feedback). Lastly, students record their progress from each trial as a measure of on-going progress (i.e., progress monitoring).

The research on MTM has been mainly compared to other math computational fluency interventions because of its novel approach. In particular, MTM has been compared to cover-copy-compare and to taped problems and has been shown to be at least as effective for the elementary participants in the studies (i.e. Mong & Mong, 2010; Mong & Mong, 2012). For example, Mong and Mong (2012) examined the effects of MTM, CCC, and taped problems on math performance of three elementary students with basic math skills deficits. The targeted skill for all participants was double-digit addition problems with regrouping (i.e. 24 + 36). Although gains in DCPM were observed across all three interventions when compared to baseline, two of the three participants produced higher gains during the MTM treatment than when provided CCC and taped problems.

Despite the effectiveness of MTM, Mong and Mong (2012) reported that this intervention may not be as efficient when compared to other computation fluency interventions such as cover-copy-compare (CCC). In both their studies (i.e., Mong & Mong, 2010; Mong & Mong, 2012), efficiency measures used by the authors suggested that MTM took twice as long to implement compared to CCC. Additionally, when compared to the taped problems intervention, MTM was reported to take three times as long, therefore practitioners may opt to implement the more efficient intervention given that instructional minutes are limited.

Incremental Rehearsal

Incremental rehearsal (IR) is a drill rehearsal intervention that involves gradually increasing the ratio of unknown to known facts until the student achieves fluency with all unknowns (Burns, 2005). To successfully implement IR with multiplication facts, an interventionist must first assess the student's skill in multiplication facts and establish two piles of facts, known and unknowns. The interventionist then selects 9 known facts and 1 unknown to begin the first round of treatment. The interventionist then introduces the unknown fact followed by the first known fact. The unknown fact is then reintroduced followed by the second known fact. This process is repeated until all knowns are cycled through and the unknown then becomes the first known and a new unknown is introduced using the same procedures.

IR has led to improved multiplication fact fluency among elementary students with generalized performance to multiplication fractions and word problems (Burns, 2005; Codding et al., 2010). For example, Burns (2005) conducted a study examining the effects of IR on single digit multiplication facts for three students with a learning disability. Results indicated an increase in performance over baseline with no overlapping data. Despite the effectiveness of IR, a disadvantage is that it has been implemented and researched only as a one-on-one intervention approach (Burns, Dean, & Foley, 2004) whereas other acquisition and fluency-based interventions (e.g. cover-copy-compare) can be implemented within small groups.

Reward

A school-based intervention that can feasibly be implemented in classrooms to increase motivation and improve accurate task completion is contingent reward (Holt, 1971; Taffel & O'Leary, 1976). Delivering a preferred reward contingent on meeting a specified task criterion, often set at 30% above baseline performance (Jones & Wickstrom, 2002), has been shown to be effective for improving academic performance.

For example, Codding et al. (2009) reported the results of BEAs that tested math interventions. For one participant, providing an incentive and subsequent reward to improve performance on single digit multiplication problems was more effective for improving fluency than other strategies that did not involve skill instruction (i.e., performance feedback, goal setting). Furthermore, the authors analyzed the incentive intervention for its effectiveness during an extended analysis and confirmed that the BEA-identified intervention (incentive/reward) supported the participant's single digit multiplication performance towards mastery. Although providing contingent reward has been shown to be effective with some individuals, unknown is for whom and under what conditions contingent reward leads to improved math performance.

Brief Experimental Analysis

Brief experimental analysis (BEA) is a viable method to quickly identify academic interventions for struggling students. This brief process allows an interventionist to evaluate the effects of multiple academic interventions and to select the intervention that produces the greatest improvement in performance to be prescribed and implemented (McComas et al., 2009). Interventions are assessed within a single-case experimental design in which two or more interventions are implemented alternately across sessions. Each intervention session can last approximately 20 min and one to three sessions are typically conducted per day. BEAs can be completed in as few as two days. Each intervention represents a condition and is typically implemented more than once to allow for replication of effects across sessions within the condition. Sessions are conducted by an interventionist who also administers a brief skill assessment following each session to measure the dependent variable (e.g., number of words read correctly, digits correct per min). Given that the unit of analysis is student-specific performance within and across instructional approaches, single-subject experimental designs are used to demonstrate experimental control of the effects of each approach. These analyses are conducted within relatively brief periods of time because they are designed to be prescriptive.

Coolong-Chaffin and Wagner (2015) describe three steps of BEA procedures along with an illustrative example (see Figure 2). The first step is to collect baseline data by having the student perform a task (e.g., reading three different passages for one minute each) while the interventionist counts the total number of words read correctly for each passage. The second step is to briefly test a series of interventions that are matched to the hypothesized reason for poor performance (e.g., the student is not motivated, the student has not practiced the skill enough) and identify the intervention that produces the highest score. The last step is to examine the effectiveness of the intervention that produced the highest score by continuing to implement the specified intervention over a period of multiple sessions. Ultimately, this process allows the interventionist to identify an effective intervention tailored to the need(s) of an individual student. The utility of BEAs has been studied far more extensively in the area of reading than math.

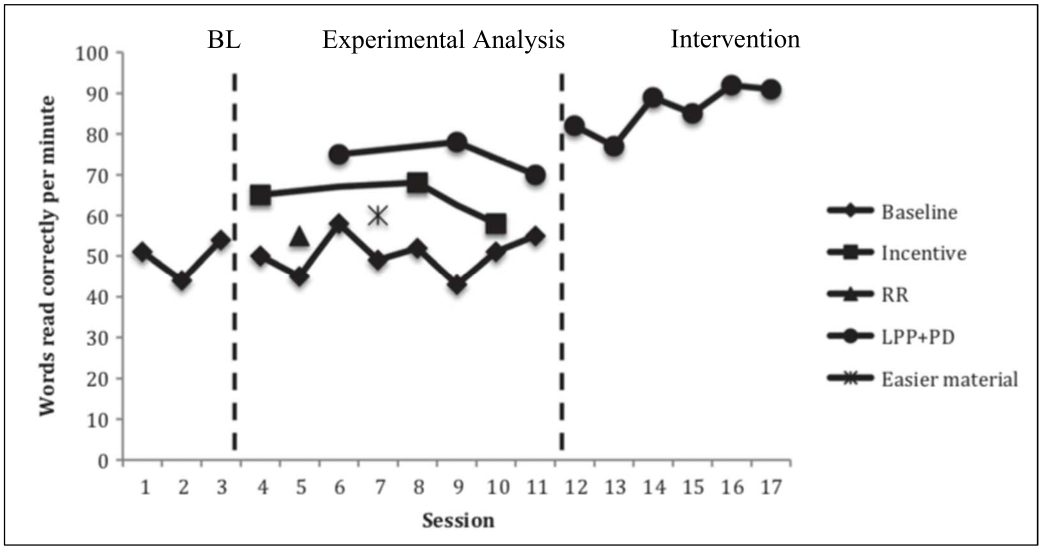

Figure 2. Illustrative example of a BEA for reading (Coolong-Chaffin & Wagner, 2015). Baseline data are collected in Phase 1. Multiple interventions are tested in Phase 2. The intervention producing the highest score is further evaluated in Phase 3.

BEA for Reading

BEA of academic interventions has been investigated and implemented since the 1990s, primarily with elementary students struggling with reading. Researchers such as Daly, Martens, Hamler, Dool, and Eckert (1999); Eckert, Ardoin, Daisey, and Scarola (2000), and McComas and colleagues (2009) have used BEA to identify reading interventions that were effective in supporting elementary students' oral reading fluency. For example, Daly et al. (1999) investigated the effects of multiple reading interventions (e.g., reward, repeated reading, repeated reading with sequential modification, and listen passage preview with repeated reading) shown to improve reading performance for struggling readers. The study consisted of testing these interventions on oral reading fluency of four elementary students. By using the BEA process as previously described, the researchers found that at least one intervention improved the oral reading fluency rate of each participant as measured by the number of words read correctly per minute.

BEA for Math

Although the aforementioned scholars have demonstrated the effectiveness of BEA to evaluate multiple reading interventions for elementary students, relatively few studies have utilized BEA to assess interventions for math or for older students (Codding et al., 2009; Reisener et al., 2016). In a recent study, Mong and Mong (2012) conducted a BEA to evaluate the effects of three different math interventions (i.e., math to mastery, cover-copy-compare, taped problems) on computational fluency with multi-digit addition problems for three elementary students. Following the identification of the most effective intervention from the BEA, the authors conducted an extended analysis of the same interventions. Specifically, the extended analysis employed an alternating treatments design in which interventions were implemented across several sessions to evaluate and confirm results from the BEA. The extended analysis phase lasted five weeks and results confirmed the effects obtained from in the BEA. Findings from Mong and Mong (2012) led the researchers to suggest that BEA is an effective assessment process that predicted an effective math intervention to improve math fluency for the students in their study.

Additionally, Reisener et al. (2016) conducted a 2-part investigation employing a BEA to evaluate the effects of different math interventions shown to improve multiplication fact fluency (i.e., Constant Time Delay, cover-copy-compare, reward) with eight participants. The authors used a multielement design during the BEA to evaluate the effectiveness of each intervention condition on the number of digits correct per minute. Following identification of the most effective intervention tested in the BEA, the authors proceeded to evaluate the effects of the most effective against the least effective intervention by conducting an extended analysis to compare the two interventions. Further, the authors included a verification phase following the extended analysis where they conducted the effective intervention in isolation in order to verify its effectiveness as demonstrated in the BEA. The authors reported that BEAs effectively prescribed an effective math intervention within a Tier 3 Response to Intervention (RtI) framework for supporting individual academic needs in math (e.g., multiplication fact fluency).

BEA can be a viable method to support students' accuracy/acquisition and efficiency/fluency needs within a tertiary level of instructional support by helping to identify an effective individualized intervention to improve performance of basic math facts. As the number of BEA studies evaluating math interventions is limited, there are still a number of unknowns regarding the use of BEAs to identify effective math interventions, especially for older students who struggle with math problems (Reisener et al., 2016).

Response to Intervention

Response to intervention is a prevention and intervention framework consisting of multiple components (see Figure 3) used to identify, evaluate, and support students with academic needs (Fuchs & Fuchs, 2007). The components of an RtI framework include (a) accurate universal screening to ensure that all students at risk for academic difficulties are identified as early as possible, (b) valid, frequent, and reliable progress monitoring that measures a student's responsiveness to the intervention, (c) data-based decision making using student data, and (d) a multi-tiered support system with a minimum of three distinctive tiers that incorporate evidence-based interventions (Bradley, Danielson, & Doolittle, 2007; Fuchs & Fuchs, 2006; Fuchs & Vaughn, 2012; Jimerson, Burns, & VanDerHeyden, 2016; Vaughn et al., 2008). These components are the foundational features of an RtI framework that can support students struggling with academic needs.

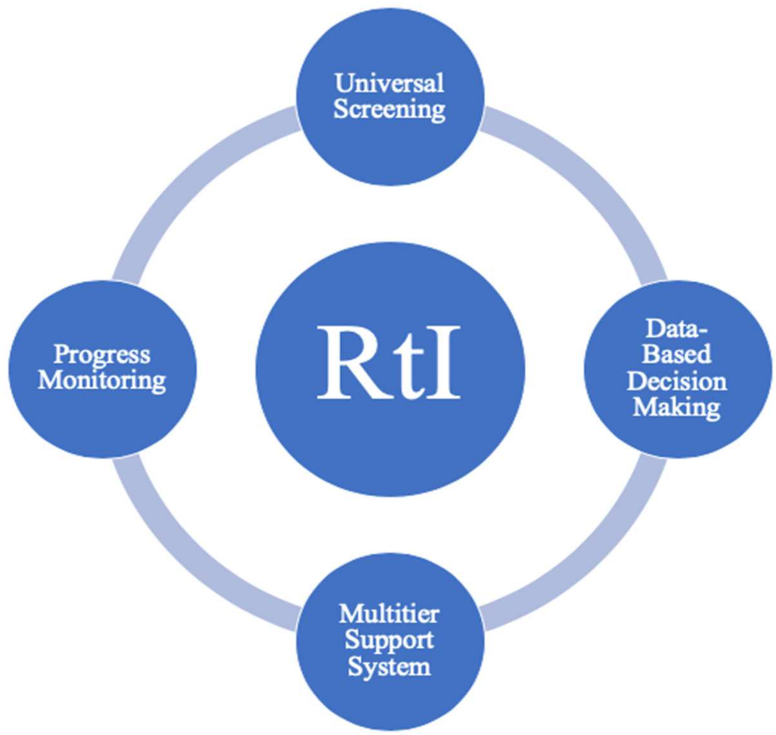

Figure 3. Core components of the Response to Intervention framework.

Universal Screening

Universal screenings are brief assessments that are reliable, valid, and predict proficiency on annual standardized assessments. It is recommended that screening be administered to all students in every grade (National Center on Response to Intervention [NCRTI], 2010). The purpose of universal screening is to identify students who are at risk of inadequate learning outcomes and therefore may be in need of more intensive intervention. Screenings are typically administered three times per year, usually in the fall, winter, and spring. One example of a brief assessment tool that can be used for screening is curriculum-based measurement ([CBM]; Deno, 1985).

In contrast to screening students in reading where screening measures are usually one-minute timed assessments to measure oral reading fluency, mathematical CBM screening measures can range between one and eight minutes and can be constructed to measure a variety of different math domains across grade levels. For example, pre-K students will be screened for early numeracy whereas elementary students will be screened on computation, concepts, and application; higher grades will focus on estimation and algebraic equations (Clark & Shinn, 2004; Chard et al., 2005; Lembke & Foegen, 2009).

Progress Monitoring

Progress monitoring is the repeated measurement of student performance that can be used to track growth and inform instruction (NCRTI, 2010). Students who have been screened and identified as at-risk are usually progress monitored. The purpose of progress monitoring is to track students' rate of growth and their responsiveness to the provided intervention. Progress monitoring should occur on a weekly basis to acquire a frequent indication of how the student is performing (Fuchs & Fuchs, 2007). Curriculum-based measurement (CBM) of math computation is an example of how students can be progress monitored measuring their computation fluency of basic math facts (Fuchs et al., 2007). Students are given a page of 25-30 math facts (e.g. multiplication) in random order and are allotted two minutes to complete as many facts as possible. Digits correct per minute are recorded as the dependent measure. Similarly, curriculum-based measurement of word problem solving (CBM-WPS) is another form of math CBM that assesses a student's word problem solving performance in mathematics (Jitendra, Dupuis, & Zaslofsky, 2014).

Data-based Decision Making

Data-based decision making incorporates the use of student data to make instructional decisions that impact the educational success of students. Graphing a student's performance on progress monitoring probes over time and comparing the trend to a goal line can greatly improve the data-based decision-making process and can ensure that the best instructional decisions are made to promote a successful academic experience for students. According to Riccomini and Witzel (2010), if a student's growth is not apparent or if the data trend on a graph appears to decline compared to the goal line, one interpretation is that the particular intervention is not effective for promoting adequate academic growth. If student data show inadequate growth, a teacher can make adjustments to their instructional procedures or interventions.

Tiers of Support

A major component of an RtI framework is the multi-tier support system in which students are provided with increasing levels of support as needed as they move up the tiers (Jimerson, Burns, & VanDerHeyden, 2016). The purpose of this system is to provide early intervention for students who experience early signs of learning difficulties before they are referred for a comprehensive special education evaluation (Xu & Drame, 2008). School districts that have chosen to implement RtI may have four or more tiers depending on their needs (Fuchs & Fuchs, 2006; Klingner & Edwards, 2006) but the most commonly used system incorporates a three-tier approach.

The first tier of a multitier support system consists of general education instruction, often referred to as core instruction (Haager, Calhoon, & Linan-Thompson, 2007). All students should receive core instruction, which is provided to ensure that students are presented with high quality and rigorous instruction to teach fundamental content and stimulate critical thinking skills. Quality core instruction should be evidence-based and aligned with state standards. Because universal screening and progress monitoring occurs within this tier, the data obtained from these brief assessments can assist the teacher in providing effective instruction. When teaching mathematics at this first tier, core instruction should be differentiated and students should be provided with flexible grouping and opportunities to peer tutor each other based on screening and progress monitoring data (Lembke, Hampton, & Beyers, 2012). In comparison to the extensive literature on RtI in the area of reading, there is somewhat limited research on RtI in the area of math, thus Gersten et al. (2009) reported that there has not been much consensus on the specifics of math instruction in this first tier except that instruction should be high quality. It is the belief that within this initial tier of a multitier support system, high quality core instruction be explicitly provided to every student, although 20% of the total students in a school building may require supplemental support at Tier 2, Tier 3, or both (Vaughn & Fletcher, 2012).

Students who fall below grade level expectations during screening may require Tier 2 support. The second tier of a multi-tier support system usually consists of student groups (e.g. 3-5 students) with similar instructional needs (Linan-Thompson & Ortiz, 2009). These small student groups are provided with evidence-based interventions to target their instructional needs and to supplement the quality core instruction they receive in Tier 1. Within Tier 2, it is suggested that math interventions focus on improving mathematical skills and competency and thus, instructional delivery of math interventions at this tier must be explicitly and systematically taught, allowing for teacher modeling, guided practice, and corrective feedback (Gersten et al. 2009).

Tier 2 also involves progress monitoring that is used to track the students' level of response to the intervention being provided. In addition, it allows for the opportunity to obtain data, which can be used for decision making and justifying the need for particular instructional support or movement of students between tiers (Jimerson et al., 2016; Orosco & Klingner, 2010). Researchers recommend that progress monitoring should occur at least once per week in order to ensure an accurate measure of students' rate of learning (Fuchs & Fuchs, 2006). Scholars suggest that approximately 15% of students will require the provision of small group intervention at Tier 2.

Tier 3 of a multi-tier support system is reserved for the approximate 5% of students who have been appropriately screened and have received supplemental instruction, yet their progress monitoring data indicate that they require more intensive and individualized instructional support to be successful in one or more content areas (Linan-Thompson & Ortiz, 2009; Gersten et al. 2009). Examples of intensive individualized support may include a smaller student to teacher ratio (3:1) or providing one-on-one instruction using evidence-based interventions. In many cases, Tier 3 interventions are highly individualized, tailored to the specific academic needs of the individual student, and can be identified via a BEA that examines the effect of instructional components on individual student academic performance.

Progress monitoring remains critical in tier 3 as it is used to evaluate student progress and inform instructional decision making (Fuchs & Fuchs, 2006; NCRTI, 2010). It can also be used to identify instructional strengths and weaknesses, adding an opportunity to tailor instruction to the needs of individual students. The premise of an RtI framework is that struggling students will progress through the tiers, receiving the appropriate level of intervention to support their improvement and mastery towards grade level expectations and then if successful, fade and eventually discontinue the supplemental interventions or support services.

Chapter III

Method

Setting and Participants

Five middle-school participants were selected from general education classrooms in a public school in the Midwest; four participants completed all study procedures. The school is located in an urban community and serves 944 students in grades K-8. Approximately 62.3% are English learners with 95.7% receiving free or reduced lunch. The majority of students are Hispanic or Latino (55.1%) followed by African American students (36.5%), American Indian (3.6%), Caucasian (2.4%), and Asian (1.2%). Participants were referred by the school's multi-tiered system of supports (MTSS) team due to their low performance in grade-level mathematics. Criteria for participation (see Table 2) included (a) scoring below the 30th percentile on the district's quarterly math benchmark via Formative Assessment System for Teachers (FAST), (b) scoring below 30 problems correct per minute on a one-minute multiplication fact screener, and (c) scoring a level 6 or below on the district's conceptual placement value (CPV) assessment.

Table 2

Criteria for inclusion of participants
* Denotes English learner status

Participant Name	Grade	Age	Below 30th percentile	PCPM on Screener	CPV Level
Macy*	8th	14	Yes	3 PCPM	2
Lisa*	8th	14	Yes	6 PCPM	2
Penny	7th	13	Yes	4 PCPM	5
Ana*	8th	14	Yes	2 PCPM	3
Laura	6th	12	Yes	2 PCPM	6

Student participation was contingent on signed parental consent and signed student assent (see Appendix A, B). Lisa and Macy were both fourteen-year-old Hispanic female students in eighth grade. Ana was a fourteen-year-old Somali female student in eighth grade. Penny was a thirteen-year-old African American female student in the seventh grade. Laura was a twelve-year-old African American female student in the sixth grade. Laura withdrew from the project before the study was complete, thus only four participants completed all study procedures. Participants were general education students who have not been referred for special education services nor identified as having a specific learning disability in mathematics.

Sessions were conducted approximately four times per week for an average of 15 minutes each during non-core instructional time in a quiet location outside of the classroom. One interventionist conducted all screening, assessment, and intervention sessions and collected permanent product data of student performance on assessments. The interventionist was a Ph.D. student in Special Education who held a special education teacher's license, a school administrator's license, and a Master's degree in Special Education. A second Ph.D. student in Special Education, who also held a special education teacher's license and a Master's degree in special education coded interscorer agreement on the dependent variable and coded procedural fidelity both in-situ during sessions or after sessions from recordings of experimental sessions.

Materials

Screeners

A screening assessment of 40 randomly selected single digit multiplication facts consisting of numbers 0-9 was developed to screen students for participation in this study. Facts on this screener only appeared once and inverse facts were excluded. This screener consisted of problems printed in a vertical top-down fashion within 8 rows of 5 columns. Space was provided at the top of the screener for students to write the date. The second screener was the district adopted conceptual place value (CPV) assessment with limited to no psychometric properties. This is an untimed assessment consisting of 8 tasks and measures a student's level of mathematical number sense, problem solving, and algebraic thinking. Performance rating on this assessment ranges from 1 to 7 with seven being the highest level of performance.

Multiplication Flashcards for Initial Fact Assessment

One hundred flash cards (2 in x 4 in) containing single digit multiplication facts (e.g. 2 x 5, 8 x 3) using numbers 0 to 9 were used during the initial fact assessment to identify each participant's known and unknown multiplication facts. Each card consisted of one fact printed in a vertical, top-down fashion on one side with the other side showing the problem and the answer.

Intervention Materials

Intervention materials consisted primarily of worksheets and flashcards for intervention sessions during the BEA and Extended Analysis. To equally distribute unknown facts across conditions, unknown facts derived from each participant's fact assessment were randomly assigned to conditions. For each participant, the full set of unknown facts was sorted to remove all inverse facts. Next, fifteen unknown facts were randomly divided into three sets (i.e. sets A, B, C). Each set was randomly assigned to a condition (e.g., set A to MTM, set B to IR, and set C to reward). See Table 3 for the facts assigned to each condition of the BEA for each participant. A worksheet with three different versions (i.e. version A, version B, version C) was created for each condition using the facts assigned to that condition. Each worksheet contained a total of 30 facts arranged in six rows of five columns with each of the unique six facts repeated five times. The IR condition did not involve worksheets; instead, flash cards from the initial fact assessment were used during intervention sessions. In addition, a stopwatch for timing assessments and a box of rewards (e.g., pencils, erasers, fruit snacks) were used during intervention sessions. The rewards were not included in the extended analysis.

Table 3

Participants' sets of facts for each condition during the BEA

Participant	MTM	Reward	IR
Macy	6x8, 3x8, 9x4, 6x6, 9x5	3x6, 6x9, 7x7, 8x7, 7x4	8x5, 3x7, 4x8, 5x6, 9x7
Lisa	8x9, 6x3, 9x3, 7x6, 7x8	9x6, 5x5, 2x4, 2x6, 4x4	2x8, 9x9, 6x6, 3x4, 2x7
Penny	7x7, 6x6, 7x9, 4x9, 6x9	6x3, 7x8, 9x3, 8x4, 6x8	9x9, 6x7, 8x8, 7x4, 8x9
Ana	3x9, 5x2, 9x4, 6x4, 7x8	7x4, 7x2, 8x5, 3x5, 4x3	5x4, 9x2, 7x7, 8x6, 6x2
Laura	8x9, 3x8, 4x9, 7x7, 4x4	6x6, 6x8, 6x9, 8x8, 6x7	3x7, 7x4, 7x9, 8x4, 7x8

Intervention Fact Assessments and Maintenance Probes

A one-minute fact assessment was administered after every intervention session, during both the BEA and the extended analysis to evaluate participants' response to the implemented intervention. The fact assessments consisted of corresponding facts addressed during the intervention session. Fact assessments consisted of 30 facts per page with each fact repeated at least five times, and three separate versions of each assessment were created to be randomly selected for use within each condition. Following the BEA, unknown facts from each participants' BEA intervention conditions were reshuffled and assigned to a set (e.g. set A, set B, set C) to be used during the extended analysis. See Table 4 for the intervention and generalization facts assigned to each leg of the multiple probe for each participant's extended analysis. These newly assigned facts were used to construct assessments containing 30 facts arranged in six rows of five columns with each fact repeated at least five times. Each assessment had three different versions that were randomly selected for use following each intervention session of the extended analysis. The fact assessments also served as maintenance probes to evaluate participants' fact maintenance following termination of intervention.

Generalization Assessments

Generalization assessments addressed similar facts as the corresponding intervention assessment which included problems designed to assess "near" and "far" generalization (Perkins & Solomon, 1992). Near and far generalization probes consisted of 30 problems each. Three different versions (forms) containing the same problems but presented in different order were created. The near generalization assessments consisted of inverse multiplication facts that were not taught in the intervention (i.e., if 4x9 was taught in the intervention, the generalization inverse problem was 9x4.) The far transfer generalization assessments contained single digit division facts from 0-9 that corresponded to the multiplication facts addressed in the instructional set and were written in a horizontal fashion (e.g., if 4x9 was taught, 36/9 was a generalization division problem).

Intervention Survey

A seven-item intervention survey (see appendix D) based on the Children's Intervention Rating Profile (Turco & Elliot, 1986) was used to measure the social validity of this study, particularly on the independent variables used by participants. The purpose of this survey was to give participants a voice to express their opinion about their prescribed intervention. Using a Likert scale from 1 (strongly disagree) through 6 (strongly agree), participants can mark their responses on 7 items that measures the acceptance, practicality and effectiveness of their prescribed intervention.

Dependent Measure

The number of digits correct per minute is often used to measure math fluency and allows for partial credit when a correct digit is written in the correct place (Shinn, 1989). However, responding correctly to multiplication facts requires a single response that may be one or two digits. For example, 3 x 4 = 12. If participants answered 11 and we scored digits correct, they would have one digit correct despite answering the problem incorrectly. A more stringent criterion for math facts (e.g., those problems that should be answered "automatically" without the need for any computation) is the number of problems correct per min (PCPM). Measuring PCPM is a more realistic fluency objective as it is a prerequisite for student success when learning more complex math skills. Further, scholars have suggested that students who are able to compute 30-40 fact problems correct per minute continue to accelerate their rates as math tasks become more challenging (Miller & Heward, 1992; Haughton, 1972). For example, data collected by Haughton (1972) have shown that students who correctly solved single digit multiplication facts (30-40 PCPM) were able to correctly solve more complex double- and triple-digit multiplication problems (e.g. 23x42; 345x63). Thus, PCPM served as the dependent variable for this study so as to prepare students for increasing complex math problems. At the end of each session, intervention effects were assessed by measuring

PCPM using a one-minute timed assessment similar to the fact assessment described above. If participants completed all problems on the assessment, the interventionist was readily available to provide the participant with a second page and instructed them to keep working for the remainder of the minute. Every participant was given a second page of assessment at least once during the extended analysis. The performance criterion throughout the study was set at 30 PCPM, a criterion that meets the recommended 30 or more PCPM as suggested by Haughton (1972) and Mercer and Miller (1992).

Experimental Design

An alternating treatments design was used in the BEA to compare the effects of math intervention conditions (i.e. MTM, IR). A third condition, Reward, was used to test for skill versus performance deficit (Duhon et al., 2004). The order of intervention conditions was quasi randomized during the BEA for each participant. That is, one session of each intervention was randomly selected to be conducted until all were conducted once, then this procedure was repeated, ensuring that no ntervention was conducted twice consecutively. A minimum of two sessions per condition were conducted but if results were undifferentiated during the BEA, then additional sessions were conducted to determine the more effective intervention. As a result, three of the five participants needed additional sessions to demonstrate differentiation during the BEA (see participant BEA results). The intervention condition that produced the highest PCPM during the BEA was selected for further testing within an extended analysis.

A multiple-probe design across unique sets of multiplication facts for each participant was used in the extended analysis following the identification of the most effective intervention from the BEA. Phases for the extended analysis included (a) baseline, (b) the participant's effective intervention identified via BEA, and (c) maintenance. Criteria for changing phases were as follows. To progress from the baseline phase to the intervention phase, a stable baseline of at least three sessions must be obtained. To progress from the intervention phase to the maintenance phase, participants must achieve 30 PCPM on three consecutive sessions. These criteria are repeated for subsequent sets of facts (i.e., legs) during the extended analysis. Generalization measures (i.e. inverse and division) were conducted at least once throughout every phase of the extended analysis to examine whether performance improved concomitantly with performance on targeted facts.

Procedure

The study was conducted in the following sequence of activities following university approval (see Appendix C) and receipt of parental consent and participant assent:

1. Screening
2. Fact assessment to determine known and unknown multiplication facts
3. Brief experimental analysis (BEA)
4. Extended analysis
5. Intervention survey

Screening

Students who turned in their signed parent consent forms (see Appendix A) and signed student assent forms (see Appendix B) were given both screeners to assess their knowledge of single digit multiplication facts and CPV level. For the multiplication screener (see Appendix E), students were given one-minute to complete as many problems as possible from left to right and without skipping any problems. In particular, they were given the following direction:

> "***You will have one minute to complete as many problems as you can on this page. Complete the problems from left to right without skipping around. You will not be penalized for getting the answers wrong. I cannot help you, but I will let you know when the time is up and you will need to put your pencil down. You may begin.***"

After one minute elapsed, the interventionist informed participants to ***"Stop, put your pencil down"*** collected the screener and recorded the total number of PCPM at the bottom of the screener. Based on Haughton (1972) and Mercer and Miller (1992), a criterion was set at 30 PCPM for the screener. For the CPV screener, a district CPV assessment form was used which included guided directions on what to prompt for each task and how to score students' responses. Students who scored below 30 PCPM and between levels 3 and 6 on the CPV were eligible to participate in the study.

Facts Assessment

Following the initial screening, participants were assessed on their multiplication facts from 0 through 9 using flashcards as described in the materials section. Facts were presented in random order and were considered known if participants provided a correct response within three seconds and unknown if participants did not provide a correct response or any response within three seconds. This process was repeated for a total of three trials across two days for each participant. Facts that participants answered incorrectly in 2 of the 3 trials were considered unknowns and were used as the targeted skill for that participant in the study. Unknown inverse facts were removed and reserved for the purpose of assessing generalization as described above. As a result, fifteen unknown facts from each eligible participant were randomly selected to be divided into three sets (e.g. set A, set B, set C). Each set was then randomly assigned to an intervention condition for the BEA. This process was repeated again after the BEA, and prior to the start of the extended analysis, ensuring that each participants' unknowns were equally distributed across conditions.

Intervention Facts Assessments

A one-minute pre and post assessment was conducted after every session during the BEA in order to assess intervention effects. During the extended analysis, only the post assessment was conducted at the end of every session. After the interventionist presented the assessment to the participant, he gave the same directions as those described above for the screener. The interventionist started the timer when the participant touched their paper with their pencil. After one minute, the interventionist instructed the participant to stop and put down their pencil. No feedback or intervention instruction was provided during this assessment.

BEA

The BEA consisted of three intervention conditions (i.e. MTM, IR, Reward). Conditions for the BEA were conducted in a quasi-random fashion to ensure that no condition was implemented in two consecutive sessions. At least two sessions per condition were conducted during the BEA. Each intervention session lasted no longer than five minutes. The difference of PCPM between the pre and post assessment (post assessment minus the pre assessment) was recorded for each session and graphed to identify the most effective intervention condition for each participant.

Math to Mastery. The interventionist and participant had identical worksheets as described in the materials section. The interventionist instructed the participant to observe and follow along on their worksheet as the interventionist modeled how to solve the first problem using the concrete representational abstract strategy. For example, if the problem is 3x4, the interventionist would draw a table with three squares in a column going across four rows. The digit 3 would be written after each row resulting in the number 3 written four times in a vertical fashion (e.g. addition array). The interventionist would then add all the 3's together to get the correct answer. After the interventionist completed the first problem, he instructed the participant to complete the remaining problems on the same row while the interventionist observed and provided corrective feedback. This process was repeated for the remaining rows until all the problems were solved or the five-minute session ended, whichever came first. After completing each row, the instructor gave the participant a chart on which to record their performance and monitor their progress across sessions of this condition.

Incremental Rehearsal. Single digit multiplication flash cards were used for this intervention. The interventionist randomly selected nine flash cards from participants' known facts as determined by the fact assessment and placed it in a pile on the left. Participants' five unknown flash cards were placed in the pile on the right. Then, the interventionist first presented a flash card containing the first unknown fact from the pile, stated the fact and answer, and prompted the participant to state the fact and answer. After the participant restated this first unknown fact and answer, the interventionist placed this fact down creating a “done” pile and a known fact was presented prompting the participant to state only the answer. After the participant stated the answer, the interventionist placed the flash card in the “done” pile. Next, the interventionist repeated those procedures with the two facts from the “done” pile again, followed by a new known fact which was then placed in the “done” pile. This process was repeated until all known facts were presented following the first unknown fact. After all known facts were presented following the first unknown fact, a second unknown was introduced followed by the first unknown, which was now considered a known fact, and the process continued until all unknowns had been presented in the same fashion or until the allotted five-minute session time ended.

Reward. Two assessments with 30 problems each were used for this condition. Each assessment had the same facts but in different order. The interventionist administered the assessments in the same manner as described previously. After the first assessment, the number of PCPM was recorded and a 30% increase was added as the goal for the second assessment. Next, the interventionist instructed the participant to select one item from a bag of tangibles (e.g. pencils, pens, markers, folders, erasers) or edibles (e.g. lollipops, jolly ranchers, chips) and stated that if they meet the goal of a 30% increase from the first assessment then they would get the reward they chose. Participants who reached the goal in the second assessment received their chosen reward and participants who did not meet their goal received a consolation prize (e.g. a sticker).

Extended Analysis

The intervention condition that produced the highest PCPM for individual participants during the BEA was further evaluated during an extended analysis. The purpose of conducting an extended analysis was to determine whether the intervention identified in the BEA would result in improvements of PCPM for remaining multiplication facts. Procedures for conditions being implemented during the extended analysis were identical to procedures used during the BEA with the exception of an increase in intervention time and deletion of the pre assessment during each intervention session. Initially, intervention sessions were 10 min long, but after several sessions, intervention time was increased to 15 min. If at any time the interventionist detected the participant may be experiencing fatigue because of the intensity of the one-on-one intervention, he prompted the participant to indicate that they needed a brief break so that appropriate steps could be taken (e.g., a short walk to the drinking fountain for a drink of water before resuming intervention activities). Intervention sessions were implemented at least four times per week. Maintenance and generalization measures were administered during the extended analysis using procedures identical to intervention fact assessment as described above.

Table 4

Participants' sets of facts during the extended analysis

Participant	Set	Targeted Facts	Inverse Facts	Division Facts
Macy	1	8x7, 3x8, 9x7, 6x9, 3x7	7x8, 8x3, 7x9, 9x6, 7x3	56/8, 24/3, 63/9, 54/6, 21/3
	2	5x6, 7x7, 6x8, 9x4, 7x4	6x5, 7x7, 8x6, 4x9, 4x7	30/5, 49/7, 48/6, 36/9, 28/7
	3	8x5, 3x6, 9x5, 4x8, 6x6	5x8, 6x3, 5x9, 8x4, 6x6	40/8, 18/3, 45/9, 32/4, 36/6
Lisa	1	2x4, 7x6, 8x9, 2x8, 5x5	4x2, 6x7, 9x8, 8x2, 5x5	8/2, 42/7, 72/8, 16/2, 25/5
	2	9x9, 2x7, 2x6, 4x4, 6x3	9x9, 7x2, 6x2, 4x4, 3x6	81/9, 14/2, 12/2, 16/4, 18/6
	3	6x6, 3x4, 9x3, 7x8, 9x6	6x6, 4x3, 3x9, 8x7, 6x9	36/6, 12/3, 27/9, 56/7, 54/9
Penny	1	6x7, 4x9, 8x8, 7x9, 6x6	7x6, 9x4, 8x8, 9x7, 6x6	42/6, 36/4, 64/8, 63/7, 36/6
	2	6x8, 8x9, 6x3, 7x4, 8x4	8x6, 9x8, 3x6, 4x7, 4x8	48/6, 72/8, 18/6, 28/7, 32/8
	3	6x9, 9x3, 7x7, 9x9, 7x8	9x6, 3x9, 7x7, 9x9, 8x7	54/6, 27/9, 49/7, 81/9, 56/7
Ana	1	4x3, 8x6, 6x4, 6x2, 7x4	3x4, 6x8, 4x6, 2x6, 4x7	12/4, 48/8, 24/6, 12/6, 28/7
	2	3x9, 7x2, 5x2, 5x4, 8x5	9x3, 2x7, 2x5, 4x5, 5x8	27/3, 14/7, 10/5, 20/5, 40/8
	3	7x7, 3x5, 7x8, 9x4, 9x2	7x7, 5x3, 8x7, 4x9, 2x9	49/7, 15/3, 56/7, 36/9, 19/9
Laura	1	7x7, 3x8, 8x4, 6x9, 6x6	7x7, 8x3, 4x8, 9x6, 6x6	49/7, 24/8, 32/4, 54/9, 36/6
	2	8x8, 7x4, 7x8, 4x9, 8x9	8x8, 4x7, 8x7, 9x4, 9x8	64/8, 28/7, 56/7, 36/4, 72/8
	3	3x7, 6x7, 6x8, 7x9, 4x4	7x3, 7x6, 8x6, 9x7, 4x4	21/3, 42/6, 48/6, 63/7, 16/4

Intervention Survey

The intervention survey was conducted at the end of the study, during the maintenance phase of each participant's third set of facts. The interventionist administered the survey individually to each participant and informed the participant to read the directions and to complete the survey as best they can. No further instructions were provided, and students were not given a specific time limit to complete the survey.

Interscorer Agreement

A second doctoral student from the same field scored 30% of probes from each experimental condition to assess interscorer agreement. Interscorer agreement was calculated by dividing the total number of agreements (answers to math problems scored as correct or incorrect on assessments) by the total number of agreements plus disagreements and multiplying y 100. Interscorer agreement for this study was 100% across all experimental conditions for all participants.

Procedural Integrity

The author conducted all sessions of the BEA and the extended analysis. A second doctoral student from the same field was present to collect procedural integrity data for half of the BEA sessions and half of the extended analysis sessions. The remaining half of the BEA and extended analysis sessions were recorded. A procedural integrity checklist was completed for every session of each experimental condition (see Appendixes F-I). Interscorer agreement was conducted by the author and two separate doctoral students for all of the BEA conditions, and at least 30% of the intervention sessions for each participant during the extended analysis. Interscorer agreement for procedural integrity was calculated as the percentage of procedural steps completed accurately that is, the number of steps checked by the second observer as correct was divided by the total number of steps and then multiplied by 100. Procedural integrity averaged 98.97% with a ranged of 94%-100%.

Data Analysis

Participants' data from every phase of the study were graphed. Visual analysis was used to evaluate participants' data during the BEA and extended analysis. In particular, visual analysis was employed to identify overlapping data and performance rate between conditions during the BEA. The condition with the most sessions producing the highest number of PCPM was selected as the intervention to be implemented during the extended analysis. Visual analysis was also used during the extended analysis to evaluate the effectiveness of the intervention on the targeted and generalization facts.

Chapter IV

Results

Results of both the brief experimental analysis and the extended analysis for all participants are depicted here. The results are presented by participant, with their BEA first, followed by the extended analysis. Participant's performance during the BEA is described in terms of (a) the number of sessions in each condition, (b) the effect of the condition on PCPM, and (c) overlapping of data, if any, across conditions. It should be noted that some of the participants' pre-assessment scores were higher than their post assessment scores resulting in a negative score. For the purpose of visual analysis, negative scores were graphed as ≤ 0. Performance during the extended analysis is reported by phases (i.e. baseline, intervention, maintenance) across sets of facts in terms of (a) the highest number of PCPM obtained, (b) the number of sessions within each phase, and (c) level, trend, and variability across phases. Near and far generalization measures are also reported by phases across sets of facts.

Macy

Brief experimental analysis. Three sessions were conducted in each of the three conditions of Macy's BEA (see Figure 4). The IR and Reward conditions resulted in improvement of two or fewer PCPM. By contrast, Macy's performance improved between 6 and 19 PCPM in the MTM sessions. Further, data in the MTM sessions do not overlap with data from the IR or Reward conditions. Given the superior effect of MTM compared to IR and Reward conditions, MTM was identified as the intervention for Macy.

Extended analysis. Figure 5 depicts the effects of MTM on Macy's performance. During baseline of the first set of facts (first leg of multiple probe), Macy scored 3 PCPM on the target facts in each of the three sessions. The first MTM intervention session resulted in a score of 7 PCPM. Macy reached 30 PCPM after seven intervention sessions and met criterion for moving to maintenance after nine sessions. During the maintenance phase, Macy maintained 30 PCPM for two of four maintenance probes.

Generalization probes during the baseline phase included a high score of 6 PCPM for inverse multiplication facts and a high score of 12 PCPM for division problems. During the intervention phase, Macy's performance on un-taught inverse multiplication facts and division problems increased to 20 PCPM and 18 PCPM, respectively. During the maintenance phase, four inverse probes and four division probes were conducted. Macy's performance was variable, ranging from 12 to 22 PCPM on inverse probes and 13 to 22 PCPM on division probes. She never reached the goal of 30 PCPM in either inverse or division generalization probes.

In the second leg of the multiple probe, stability was established after five baseline sessions, with performance ranging from 2 to 7 PCPM. After the first intervention session, Macy's performance immediately improved to 21 PCPM. She reached the goal of 30 PCPM in the second intervention session and after seven sessions, she met criterion for moving to maintenance. During the maintenance phase when intervention was discontinued, Macy did not maintain 30 PCPM on the target facts, scoring between 12 and 24 PCPM across four probes.

Generalization probes during baseline ranged from 4 to 9 PCPM on inverse multiplication facts and 2 to 13 PCPM on division problems. During intervention, Macy's performance on un-taught inverse facts improved slightly to 10 PCPM and her performance on un-taught division problems increased to 24 PCPM. Her performance on generalization probes during the maintenance phase did not increase any further, and she never reached the goal of 30 PCPM with either inverse facts or division problems.

In the third leg of the multiple probe, five probes were conducted on the target facts. Macy scored between 8 and 14 PCPM on the third set of target facts during baseline. Macy immediately reached 30 PCPM in the first intervention session and after five intervention sessions met criterion to discontinue intervention and move to the maintenance phase. Macy maintained more than 30 PCPM on target facts during the maintenance phase.

Generalization probes during baseline ranged from 9 to 18 PCPM on inverse multiplication facts and 10 to 19 PCPM on division problems. During intervention, Macy's performance on un-taught inverse facts and division problems remained in the same range as her baseline performance. However, during the maintenance phase, Macy's performance on both inverse facts and division problems improved; she scored 29 PCPM on all three inverse probes and 30 PCPM in two of the three division probes.

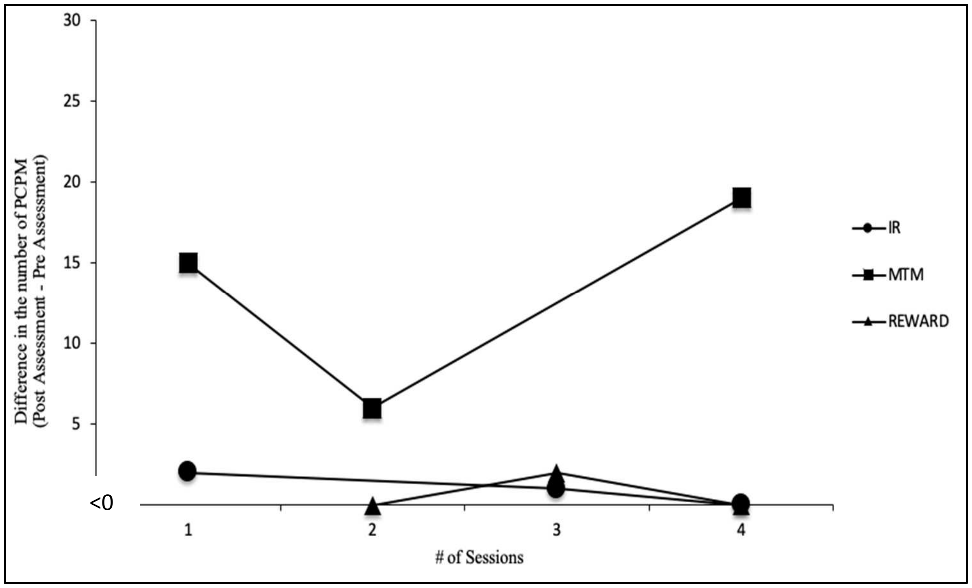

Figure 4. BEA results for Macy

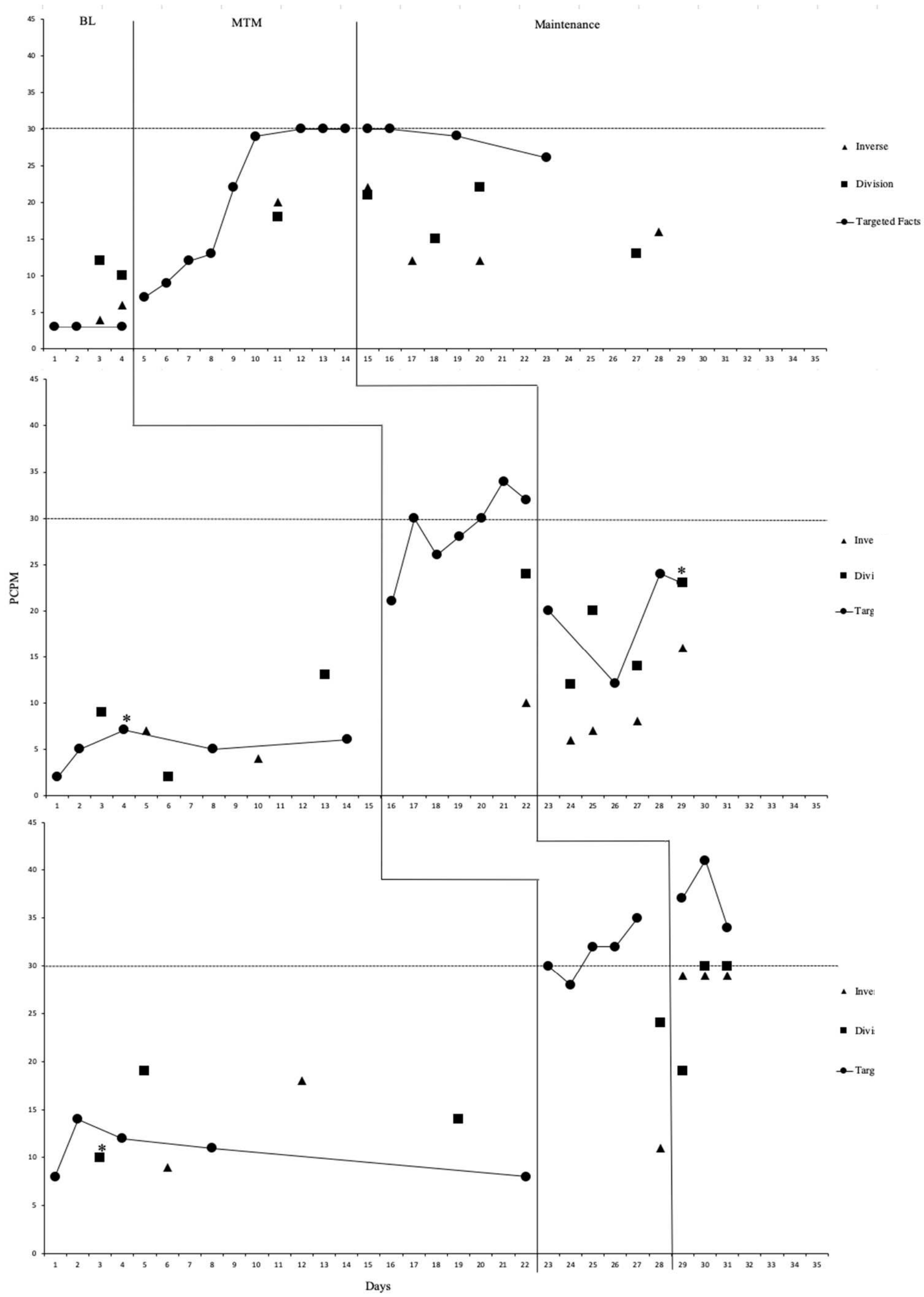

Figure 5. Extended analysis results for Macy. Dotted line denotes criterion at 30 PCPM. Asterisks represents two or more sessions with the same score.

Lisa

Brief experimental analysis. Four sessions were conducted for each of the three conditions in Lisa's BEA (see Figure 6). In the IR condition, Lisa's performance improved by 1 PCPM for her first three sessions before an improvement of 3 PCPM in the last session. In the Reward condition, she improved 2 PCPM during the first session with a decrease in performance for subsequent sessions. In the MTM condition, her performance improved by 2 PCPM in two of the four sessions and by 1 PCPM in one of the sessions, with no improvement in one session. Despite considerable overlap in data across conditions during Lisa's BEA, the MTM condition produced the greatest improvement relative to the other two conditions, so MTM was identified as the most promising intervention to improve her performance with multiplication facts.

Extended analysis. Figure 7 depicts the effects of MTM on Lisa's performance. For the first set of facts (Leg 1 of the multiple probe), three sessions were conducted with target facts during baseline and her performance reached as high as 4 PCPM. Immediately after intervention was introduced, she achieved 33 PCPM and met criterion after just three sessions. During the maintenance phase, she maintained the goal of 30 PCPM or better in three out of four sessions.

Two near and two far generalization probes were conducted during baseline of the first leg of the extended analysis. The highest score she achieved was 4 PCPM for multiplication inverse facts and she scored 0 PCPM on division problems. During the intervention phase, Lisa's performance on untrained inverse multiplication facts increased to 29 PCPM but performance on division problems remained at 0 PCPM.

Three inverse probes and three division probes were conducted during the maintenance phase and Lisa met the goal of 30 PCPM in one inverse probe but her performance on division probes remained low, scoring no more than 8 PCPM.

In the second leg of the multiple probe, four sessions were conducted with targeted facts and her performance ranged from 0 to 3 PCPM. When intervention was introduced, Lisa's performance immediately improved to 16 PCPM. After the first intervention session, she reached 30 PCPM and met criterion for moving to maintenance after four sessions with intervention. When intervention was discontinued during the maintenance phase, she met the goal of 30 PCPM once with scores ranging from 26 to 30 PCPM across four total probes.

Generalization probes during baseline of the second leg remained consistent at 3 PCPM for inverse facts and 0 PCPM for division problems. During the intervention phase, Lisa's generalization performance on untrained inverse facts improved to 26 PCPM but her performance on division problems remained at 0 PCPM. During the maintenance phase, her generalization performance on three inverse probes slowly improved and she met the goal of 30 PCPM on her last probe with scores ranging from 10 to 30 PCPM. Lisa's performance on division probes during maintenance was essentially unchanged from baseline.

In the third leg of the multiple probe, five probes were conducted on the targeted facts and she never scored above 2 PCPM. With the introduction of MTM, Lisa scored 25 PCPM during her first intervention session and reached criterion for moving to maintenance after four intervention sessions. During the maintenance phase, Lisa met the goal of 30 PCPM in two of the four probes of targeted facts.

Generalization probes during baseline ranged from 0 to 2 PCPM on inverse multiplication facts and division probes remained at 0 PCPM. During the intervention phase, Lisa's performance on untrained inverse facts improved to 21 PCPM with performance on untrained division problems improving to 2 PCPM. During the maintenance phase, Lisa met the goal of 30 PCPM during one of the three inverse probes. Her performance on division problems never reached more than 6 PCPM.

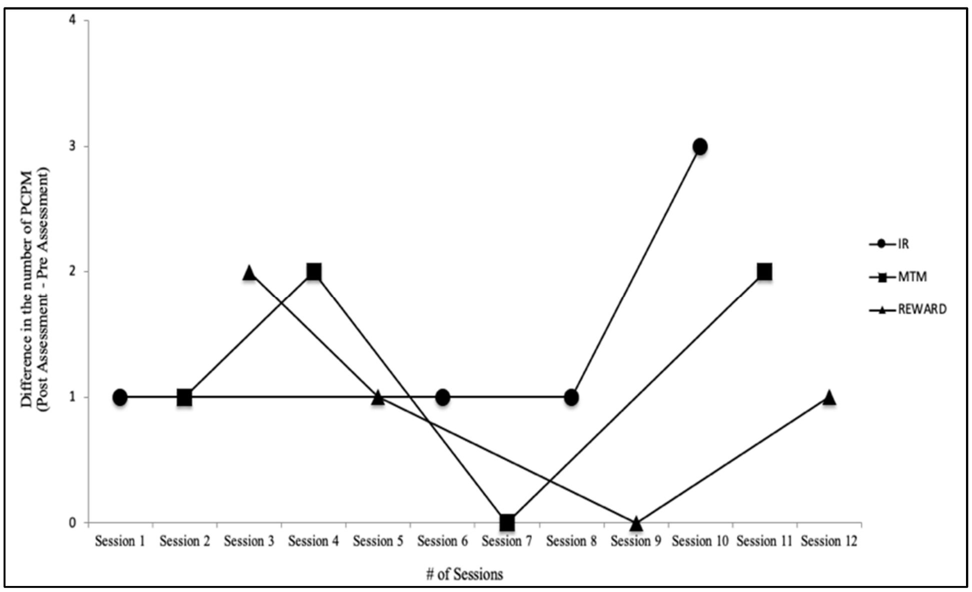

Figure 6. BEA results for Lisa

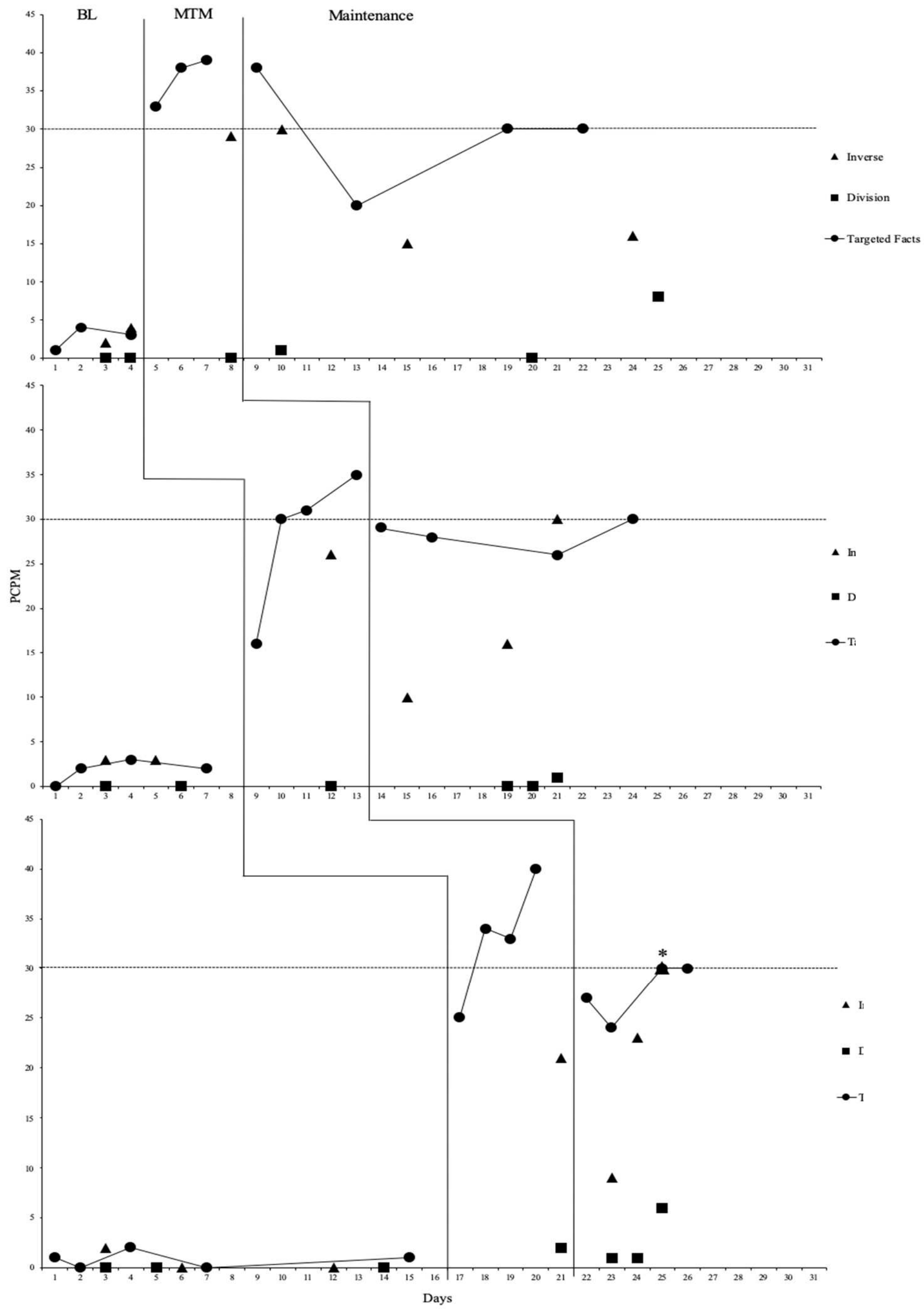

Figure 7. Extended analysis results for Lisa. Dotted line denotes criterion at 30 PCPM. Asterisks represents two or more sessions with the same score.

Penny

Brief experimental analysis. Two sessions of the IR and Reward conditions and three sessions of the MTM condition were conducted (see Figure 8). The Reward condition resulted in 4 PCMP and 0 PCPM in the two sessions, respectively. Across three MTM sessions, Lisa achieved between 0 and 5 PCPM. By contrast, her performance in the IR condition ranged from 5 to 13 PCPM. Given the superior effect of the IR condition relative to the MTM and Reward conditions, IR was the identified intervention for Penny.

Extended analysis. Figure 9 depicts the effects of IR on Penny's performance. In the first leg of the multiple probe, three baseline sessions were conducted of target facts and Penny obtained 6 PCPM or fewer in each session. After intervention was introduced, Penny's performance improved slightly to 10 PCPM. For her fourth and subsequent sessions, intervention time was increased to 15 minutes. After two additional sessions, Penny reached 30 PCPM and met criterion to move to maintenance after a total of eight sessions with intervention. During maintenance, she met the goal of 30 PCPM during one of four probes.

Two near and two far generalization probes were collected during the baseline phase of the first leg and Penny the highest scores Penny achieved were 6 PCPM for inverse multiplication facts and 10 PCPM for division problems. During the intervention phase, her performance on untrained inverse facts and division problems improved to 19 and 20 PCPM, respectively. During the maintenance phase, four inverse probes and four division probes were conducted. Penny's performance was variable, ranging from 4 to 24

PCPM on inverse probes and 16 to 30 PCPM on division probes. She met the goal of 30 PCPM in the third division probe.

For Penny's second leg of the multiple probe, five baseline sessions were conducted with the target facts and her performance ranged from 3 to 7 PCPM. It took Penny seven sessions to reach the goal of 30 PCPM and she met criterion for moving to maintenance after an additional three sessions. Penny maintained the goal of 30 PCPM for three of four maintenance probes on targeted facts.

Penny's generalization probes during baseline of the second leg ranged from 3 to 5 PCPM on inverse multiplication facts and 8 to 12 PCPM on division problems. During intervention, Penny's performance on un-taught inverse facts and division problems improved to 18 and 21 PCPM, respectively. During maintenance, her performance on inverse facts was variable ranging from 11 to 28 PCPM and she met and maintained the goal of 30 PCPM on division problems.

In the third leg of the multiple probe, six baseline sessions of the targeted facts were conducted, and Penny scored between 2 and 6 PCPM. Penny's performance immediately improved to 20 PCPM when intervention was introduced. After seven intervention sessions, she met criterion to discontinue intervention and move to the maintenance phase. Penny maintained 30 PCPM or more in two of three maintenance probes.

Penny scored between 1 to 8 PCPM on inverse multiplication facts and 2 to 13 PCPM on division problems in baseline of the third leg of the multiple probe. During intervention, her performance on generalization probes remained consistent with baseline performance. However, during the maintenance phase, Penny's performance on both inverse facts and division problems improved; she scored between 19 to 24 PCPM on three inverse probes and 22 to 30 PCPM on three division probes.

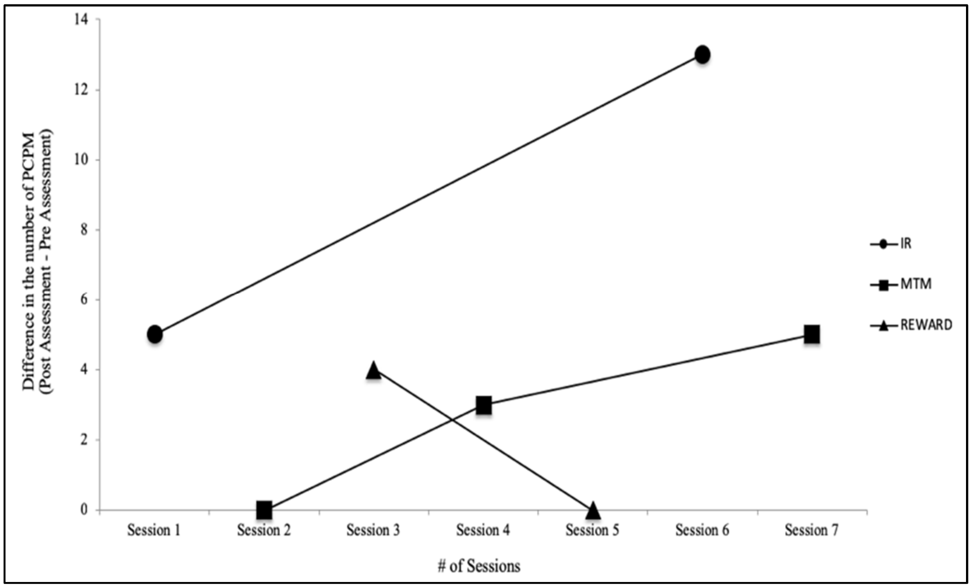

Figure 8. BEA results for Penny

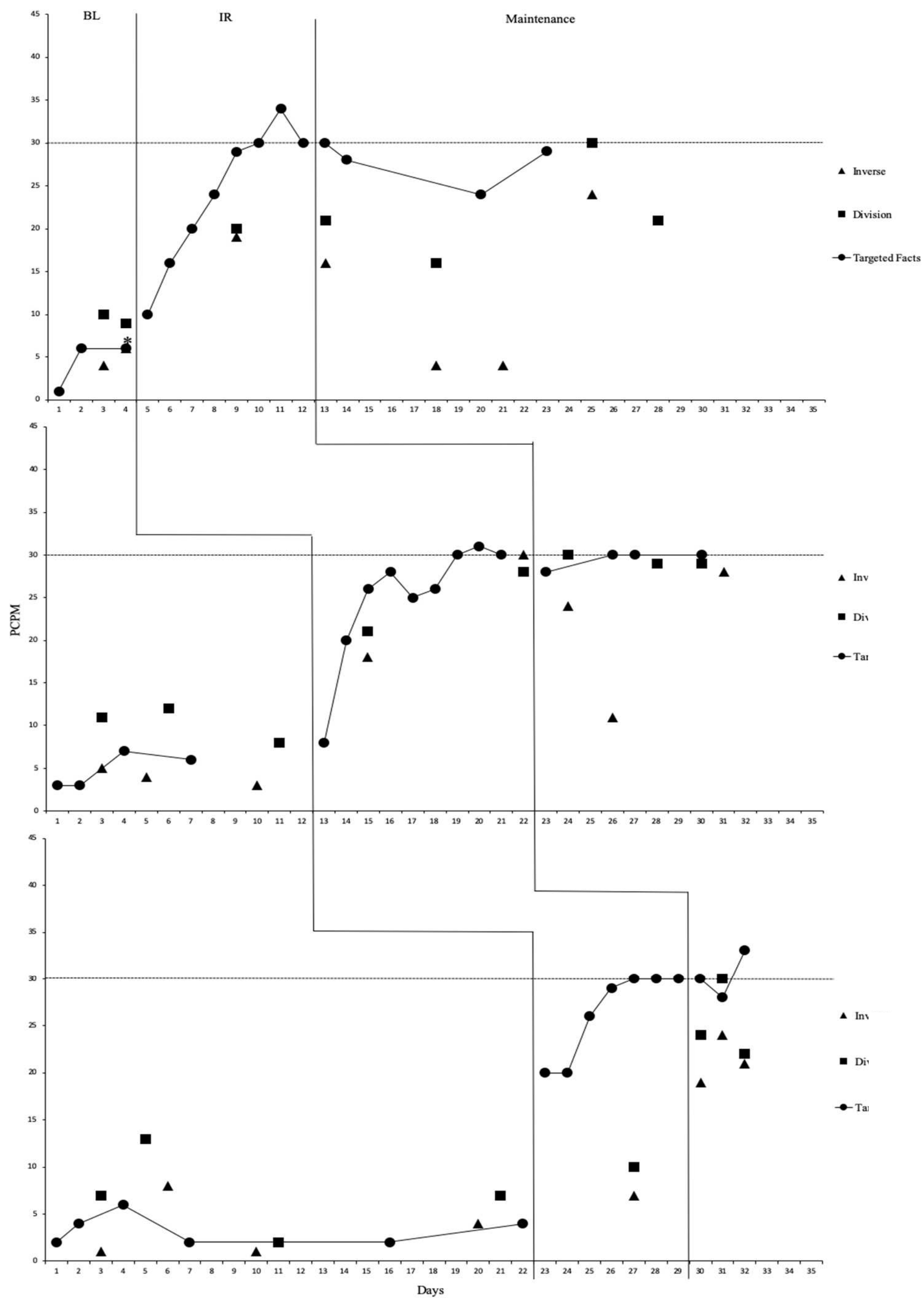

Figure 9. Extended analysis results for Penny. Dotted line denotes criterion at 30 PCPM. Asterisks represents two or more sessions with the same score.

Ana

Brief experimental analysis. Two sessions were conducted for each condition during Ana's BEA (see Figure 10). The MTM and Reward conditions resulted in improvement of 9 or fewer PCPM. By contrast, Ana's performance improved following IR between 11 and 19 PCPM. In addition, data during the IR condition do not overlap with data from the MTM and Reward conditions. Based on the superior effect of IR relative to MTM and Reward, IR was identified as the intervention for Ana.

Extended analysis. Figure 11 depicts the effects of IR on Ana's performance. In the first leg of the multiple probe, three baseline sessions of target facts were conducted, and Ana's scores ranged between 6 and 10 PCPM. The first IR intervention session resulted in a score of 24 PCPM and she reached the goal of 30 PCPM in the second session. Ana met criterion after four intervention session and her performance remained consistent at or above the goal of 30 PCPM during the maintenance.

Two near and far generalization probes were collected during baseline and she scored a high of 8 PCPM for both inverse facts and division problems. During the intervention phase, Ana's performance on untrained inverse multiplication facts and division problems increased to 24 PCPM and 16 PCPM, respectively. During the maintenance phase, three inverse and division probes were conducted. Ana's performance was variable, ranging from 15 to 29 PCPM on inverse probes and 7 to 18 PCPM on division probes. She did not obtain the goal of 30 PCPM for either the inverse or division generalization probes.

In the second leg of the multiple probe, four sessions were conducted with target facts and she scored between 2 to 10 PCPM. After three intervention sessions, Ana reached the goal of 30 PCPM and met criterion to move to maintenance after five sessions. During the maintenance phase, she maintained performance at 30 PCPM for two of four sessions.

Two near and two far generalization probes were collected in the second leg of the multiple probe, where she scored 6 and 8 PCPM for the inverse probes respectively and 11 PCPM on both division probes. No generalization probes were collected during the intervention phase. During the maintenance phase, she scored between 16 and 18 PCPM on inverse probes and 12 to 28 PCPM on division probes, never meeting the goal of 30 PCPM in either near or far generalization.

In the third leg of the multiple probe, five sessions were conducted with target facts and Ana's scores ranged from 6 to 19 PCPM. After the first session of intervention, Ana reached the goal of 30 PCPM and met criterion to move to the maintenance phase after three sessions of intervention. During maintenance, Ana maintained met the goal of 30 PCPM or better during two out of three sessions without intervention.

Three near and far generalization probes were collected during the baseline phase of the third leg and Ana's performance ranged from 9 to 19 PCPM on inverse multiplication facts and 10 to 17 PCPM on division problems. During intervention, Ana's performance on un-taught inverse and division problems improved to 24 and 27 PCPM respectively. During maintenance, Ana met the goal of 30 PCPM during one of three inverse fact probes and during all three of the division probes.

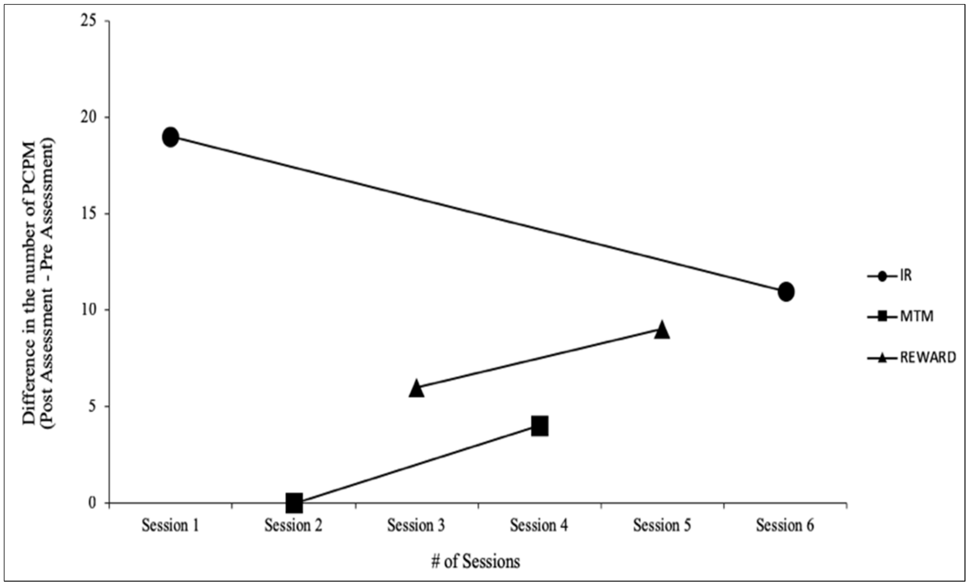

Figure 10. BEA results for Ana

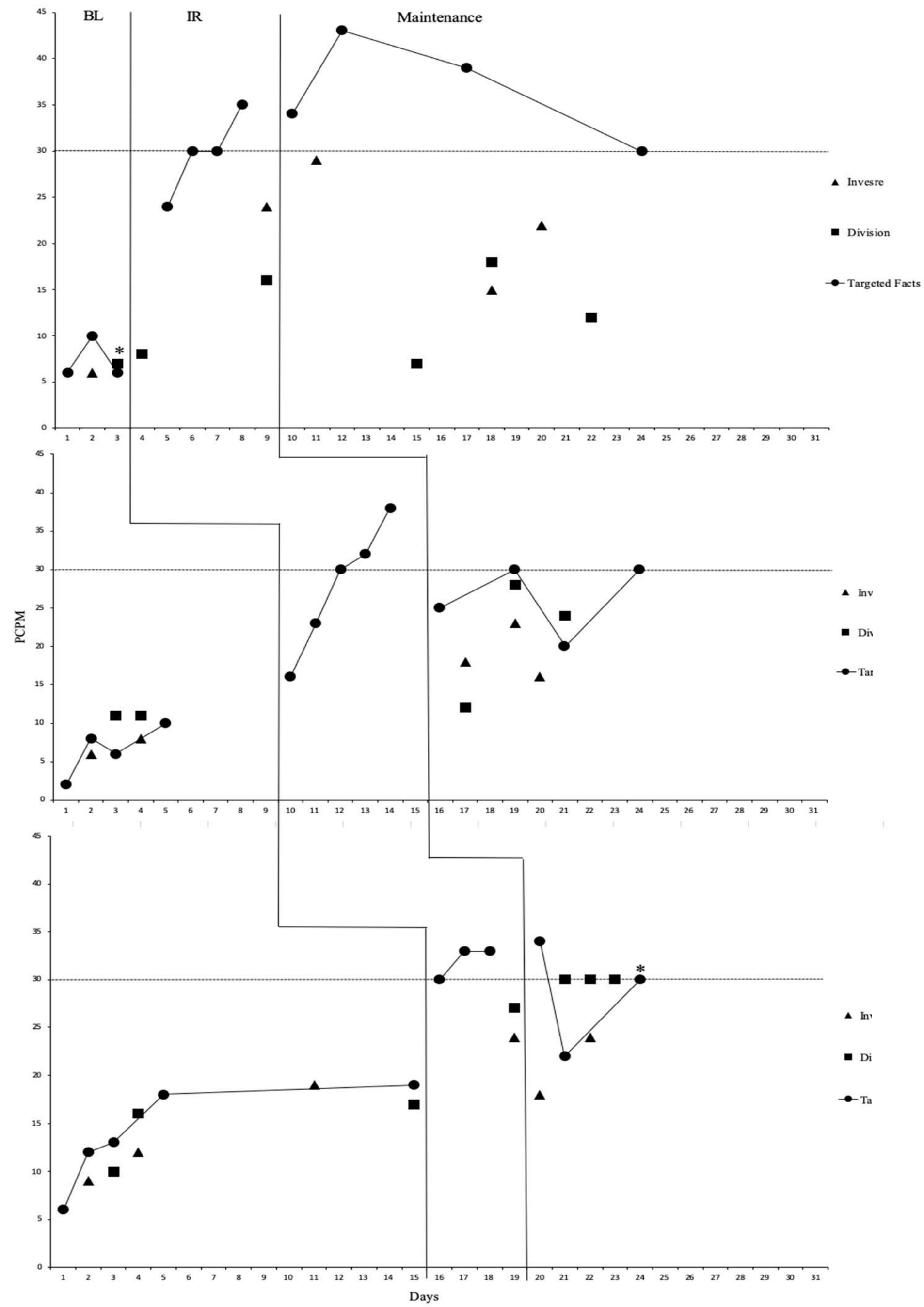

Figure 11. Extended analysis results for Ana. Dotted line denotes criterion at 30 PCPM. Asterisks represents two or more sessions with the same score.

Laura

Brief experimental analysis. Four sessions were conducted in each of the four conditions for Macy's BEA (see Figure 12). The Reward condition resulted in 4 or fewer PCPM. Both the IR and MTM conditions resulted in similar improvement overall with scores ranging from ≤ 0 to 12 PCPM for IR and ≤ 0 to 13 PCPM for MTM. Despite considerable overlap in the data of all three conditions, the MTM condition produced slightly greater gains than IR and much higher gains than the Reward condition, thus MTM was selected as the most promising intervention for Laura.

Extended analysis. Figure 13 depicts the effects of MTM on Laura's performance. In the first leg of the multiple probe, during baseline, her performance on target facts ranged from 6 to 10 PCPM across three sessions. After four intervention sessions, Laura reached the goal of 30 PCPM and met the criterion for moving to maintenance after six sessions. Laura maintained 30 PCPM in the one maintenance probe collected.

One near and one far generalization probe were collected in the second leg of the multiple probe and Laura scored 11 PCPM and 7 PCPM, respectively. During the intervention phase, Laura's performance on un-taught inverse multiplication facts and division problems increased to 27 and 14 PCPM, respectively. During maintenance, one inverse fact probe, and one division probe were conducted and Laura's performance on both was far below what it was during the intervention phase.

Limited data were obtained for Laura's second and third fact set because she began engaging in severe problem behavior at school, skipped class frequently, and eventually withdrew from the study. Thus, study procedures were terminated after one intervention session during the second leg of the multiple probe.

In baseline phase of the second leg of the multiple probe, Laura's performance on target facts was variable with scores between 3 and 8 PCPM across five sessions. One intervention session was conducted, and her score more than doubled to 17 PCPM.

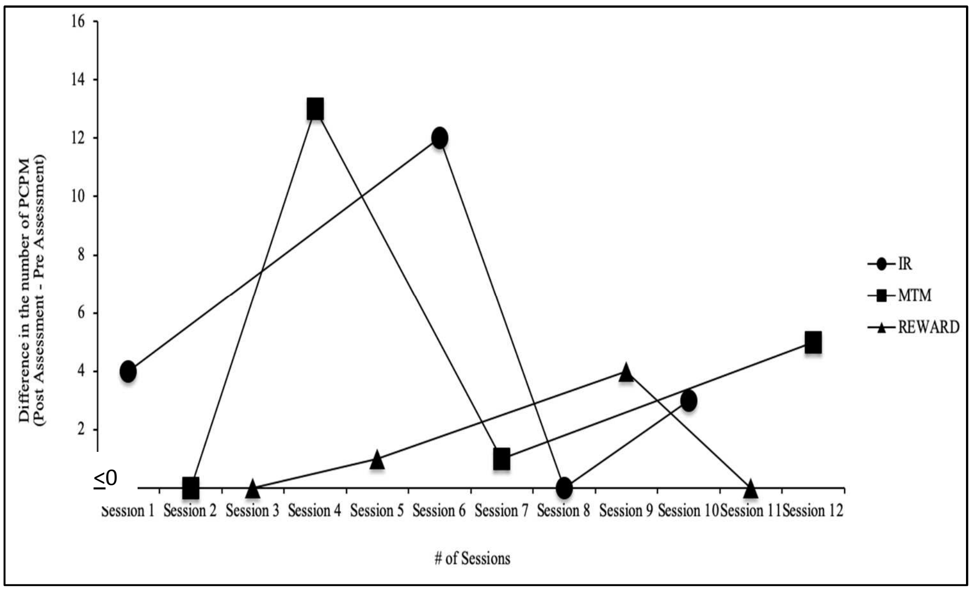

Figure 12. BEA results for Laura

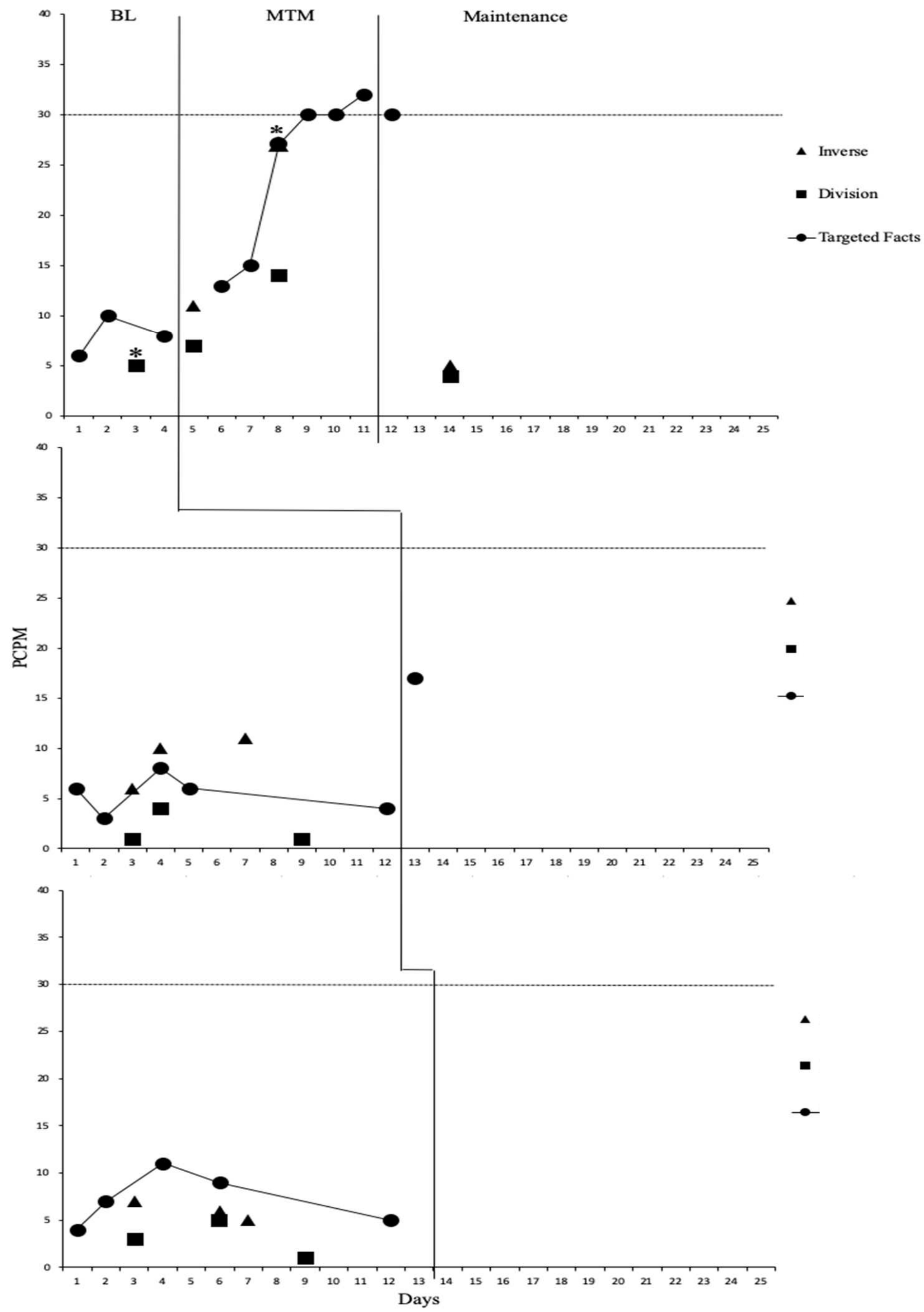

Figure 13. Extended analysis results for Laura. Dotted line denotes criterion at 30 PCPM. Asterisks represents two or more sessions with the same score.

Intervention Survey

An intervention survey was conducted to measure the social validity of this study. In particular, participants expressed their level of agreement using a Likert scale from 1 (strongly disagree) to 6 (strongly agree) on seven specific items pertaining to their BEA identified intervention. For Macy and Lisa whose BEA identified intervention was MTM, and for Penny and Ana whose intervention was IR, all strongly agreed that they liked their prescribed intervention because it is a good intervention and very helpful in teaching older students to improve their performance in mathematics (see Table 5). Furthermore, all participants strongly disagree that their prescribed intervention was too hard for them. In addition, all participants disagree that their prescribed intervention may be hard for other students. Based on participants' responses, their perception of the BEA identified intervention is acceptable, practical, and effective in improving their performance to learning multiplication facts.

Table 5

Results of the intervention survey

Questions	Mean	Range
This is a helpful way to teach math.	5.75	5-6
This way to teach math is too hard for me.	1	1-1
This way to teach math may be hard for other students.	2.25	1-4
There are other ways to teach math to students that is better than this way.	2.25	1-5
This way of teaching math is good to use with other students.	5.75	5-6
I like this way of teaching math.	5.75	5-6
I think teaching math this way will help other students learn their math.	5.5	4-6

Chapter V

Discussion

The purpose of this study was to examine the use of BEAs to identify an effective intervention to teach multiplication facts to middle school students who have not yet acquired proficiency with basic multiplication facts. Specifically, this study aimed to answer three research questions. The first question asked, "What is the effect of a BEA-identified math intervention on target math facts for middle school students who are struggling with basic multiplication facts?" The second question asked, "To what extent do gains in target facts maintain over time?" The third question asked, "What is the effect of a BEA identified math intervention on generalization to inverse multiplication facts (near generalization) and single digit division problems (far generalization)?"

The results from this study indicate that the BEA identified an effective math intervention that improved each participants' learning performance on their targeted multiplication facts (see Table 6). For three participants (i.e., Macy, Lisa, Laura), MTM was identified as the most effective intervention. For the remaining two participants (i.e., Penny and Ana), IR was identified as the most effective intervention. Reward was not identified as the most effective intervention for any of the participants, suggesting that these participants may experience skill rather than motivation issues that impacted their multiplication fact performance.

Table 6

Summary of results for each participant

Participant	BEA Identified Intervention	Fact Set #	Acquired Targeted Facts (Y/N)	Number of intervention sessions required to reach 30 PCPM & total number of sessions	Performance reached 30 PCPM or more for at least 50% of probes conducted during maintenance (M), inverse problems (IP), and division problems (DP) (Y/N)		
					M	IP	DP
Macy	Math to Mastery	1	Yes	6 required, 9 total	Y	N	N
		2	Yes	4 required, 7 total	N	N	N
		3	Yes	2 required, 5 total	Y	N	Y
Lisa	Math to Mastery	1	Yes	1 required, 3 total	Y	N	N
		2	Yes	1 required, 4 total	N	N	N
		3	Yes	1 required, 4 total	Y	N	N
Penny	Incremental Rehearsal	1	Yes	5 required, 8 total	N	N	N
		2	Yes	6 required, 9 total	Y	N	N
		3	Yes	4 required, 7 total	Y	N	N
Ana	Incremental Rehearsal	1	Yes	1 required, 4 total	Y	N	N
		2	Yes	2 required, 5 total	Y	N	N
		3	Yes	1 required, 3 total	Y	N	Y

Pertaining to the first research question about acquisition, all participants met the goal of 30 PCPM for all three fact sets after 9 or fewer BEA identified intervention sessions. All participants' target facts data show an immediate change in level with no overlap between data in the baseline and intervention phases for all fact sets indicating that when and only when the MTM intervention was introduced, performance improved.

Regarding the second research question about maintenance, all participants showed some maintenance of acquired taught multiplication facts. Judging maintenance as meeting the 30 PCPM goal for at least 50% of maintenance probes, one participant maintained acquired facts in all three fact sets and the remaining three participants maintained acquired facts in two sets. The BEA identified intervention had fewer compelling effects on generalized skills for each participant and across fact sets. Near generalization, measured in terms of inverse multiplication facts, was defined as the participant achieving the goal of 30 PCPM during the maintenance phase. Although all participants' data indicated some improvement over baseline in un-taught inverse multiplication facts in all fact sets, none achieved the goal of 30 PCPM in at least half of the near generalization probes collected during the maintenance phase. The BEA identified intervention effects were mixed for far generalization, measured in terms of related division problems. Most participants showed some improvement over baseline in division problems across the three fact sets, some showed little if any improvement over baseline, and two participants met the goal of 30 PCPM

Despite the mixed performance in generalization, data also suggest that certain participant's fluency performance on certain fact sets was sufficient for promoting generalization to inverse and division problems. For example, Macy's fluency performance during the intervention phases of the extended analysis improved subsequently from the first set of facts to the second and third sets. She met criterion during intervention with fewer sessions in subsequent fact sets. It can be hypothesized that her generalization performance in the third set of facts during the maintenance phase was facilitated by her fluency. However, fluency with one set of facts was clearly not sufficient to promote generalization and maintenance for all participants.

For participants who struggle to generalize to inverse and division problems, the BEA identified intervention can perhaps be enhanced with an intervention component designed to specifically address generalization. Alternately, it may be necessary to explicitly teach the commutative property, as in the intervention "Math Flash" (Fuchs et al., 2008).

Finally, a number of factors may be responsible for more consistent maintenance and pronounced generalization. First, students who are in middle school and who have not yet mastered multiplication facts have several years history of struggling academically which may lead to stress, anxiety, lack of enthusiasm for school-based learning, poor motivation, or any combination of these. Second, it is plausible that English language learning status may have been a factor in the mixed generalization results given that research has indicated that English learners often have language-related difficulties with math (Garcia, Lawton, & De Figueiredo, 2012).

Contributions to the Literature

Brief experimental analysis provides an efficient way to identify effective interventions for individual students (Jones & Wickstrom, 2002). The majority of BEA studies have investigated the effect of reading interventions to improve reading fluency of elementary students. In contrast to the relatively large number of BEA studies conducted in the area of reading, there is limited published research on the use of BEA to identify math interventions. Of the few published BEA studies in the area of math, virtually none have targeted basic math skills of middle school students who struggle in mathematics due to a lack of basic math skills. The current study employed BEA to evaluate multiple math interventions in order to identify an individualized effective intervention for teaching multiplication facts to middle school students.

This study contributes to the literature of using BEA to identify effective math interventions because of the similarities and differences to other BEA math studies. A few similarities include the independent variable MTM and the experimental design as used in previous BEA math studies (e.g., Mong and Mong, 2012). Therefore, this study supports the existing literature on BEAs in math. One difference in this study is the novel and practical adoption of the number of problems correct per minute (PCPM) as the dependent variable. As noted above, current math research in the field of special education primarily use digits correct per minute (DCPM) as their dependent variable because of its ability to sensitively measure growth and customize instruction but in today's world of academic expectations, high stakes assessments in math don't give half credit or points for digits placed in the correct place value. Another way this study extends the existing research is in its focus on middle school students, an older population than has been targeted in previous BEA studies. Finally, this study examined the effects of IR as an independent variable which hasn't been investigated in published research on BEAs targeting basic math facts. In sum, the findings of this study support and extend the literature on the utility of BEAs in the area of math.

Limitations and Future Research

There are a few study limitations that need to be addressed in order to inform the current literature base and to support future BEA math research. First, this study focused on multiplication fact fluency. While multiplication fluency is a necessary prerequisite to more advanced mathematics (Burns, Ysseldyke, Nelson, & Kanive, 2015), our target skill did not reflect grade-level expectations, and the generalization data indicated that the results did not consistently produce generalization, including to more advanced mathematics (i.e., division problems). Despite the use of a concrete representational abstract strategy for the MTM intervention, participants did not always automatically transfer the array of 3x4 and 4x3. Future researchers should investigate strategies or a combination of interventions (i.e. intervention package) to increase generalization from math facts to more advanced skills like division problems or grade level content materials.

Second, it is unknown whether these procedures can be implemented by current practitioners with fidelity in typical classrooms yielding similar results. All sessions in this study were conducted by one trained experimenter within a one-to-one ratio of experimenter to participant, and outside of the participants' regular classroom environment. With classroom size continuing to increase in urban settings and districts not being able to provide adequate training on certain research-based math interventions, opportunities to build teacher capacity that has a direct impact on student success is likely to continue to diminish. Future researchers should investigate the effects of teacher led

BEA in identifying an effective math intervention to support their own students with grade level content materials.

Third, this study incorporated the use of three interventions (i.e. MTM, IR, Reward) that have been supported through research to improve student performances in learning basic math facts. MTM and IR were two of the three interventions identified by the BEA as most effective for participants in this study. Considering that the Reward intervention provided participants with a tangible item which relied on extrinsic motivation and did not result in improved performance, it is plausible that participants experienced problems with intrinsic motivation which was not addressed with the extrinsic rewards offered. Intrinsic motivation may have been damaged by years of math failure and the discrepancy between their math skills and the grade-level math expectations grows. Although each intervention used in this study has a certain level of research supporting its effectiveness, there are other research-based math interventions with differential levels of effectiveness that were not evaluated in this study (e.g. taped problems, cover-copy-compare). Future research should investigate a wider range of math interventions to better understand the conditions under which intervention works best for which student.

Finally, due to the small number of participants, it was not possible to examine whether there were participant characteristics that might predict which strategies are more likely to be effective than others under particular conditions (i.e. the type of intervention, the interventionist, the number of intervention sessions and length). For example, based on findings in the BEA reading literature, one might expect that if a student can perform math problems accurately but slowly that a fluency-based intervention, perhaps involving repeated practice, might be more effective than a conceptual-based intervention such as MTM. Similarly, one might expect greater gains with MTM than a repeated practice-based intervention for a student whose accuracy is extremely low. To date, these hypotheses have not been tested. Furthermore, two participants' BEA results were not as clearly differentiated and may have required more BEA sessions or the comparison of equally effective interventions in an extended analysis. Thus, future BEA math research might aim to understand what specific interventions are likely to work under specific conditions.

Overall, results from the current study indicate that BEA efficiently identified an effective math intervention for increasing multiplication fact fluency for all five participants. In addition, results from the extended analysis suggest that the BEA-identified intervention improved fluency across different sets of facts for all four participants who completed all study procedures. Although improvement in near and far generalization tasks was observed with nearly all fact sets for all participants, there were few instances of a participant achieving the target of 30 PCPM in generalization probes. Based on the findings of this study and consistent with previous BEA math studies, the current study demonstrated that BEA is a viable method for evaluating math interventions in order to identify and prescribe an effective intervention for middle school students who have not mastered basic math facts.

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Appendix A

Parent Consent Form

Title of Research Study: ***The Effects of a Math Intervention Identified Using Brief Experimental Analysis for Middle School Students Struggling With Multiplication Fact Fluency***

Investigator Team Contact Information: ***Jennifer J. McComas***
For questions about research appointments, the research study, research results, or other concerns, call the study team at:

Investigator Name: Jennifer J. McComas Investigator Departmental Affiliation: University of Minnesota, Twin Cities Educational Psychology Email Address: jmmcomas@umn.edu	Student Investigator Name: John Mouanoutoua Email Address: mouan003@umn.edu

Key Information About This Research Study
The following is a short summary to help you decide whether or not to be a part of this research study. More detailed information is listed later on in this form.
This is a single subject experimental design study involving the use of brief experimental analysis (BEA) to quickly evaluate the effects of multiple math interventions for middle school students who struggle with multiplication facts. There will only be a maximum of 6 student participants in this study and each student will be provided with four different math interventions targeting unknown multiplication facts unique to each participant. The length of this study will consist of 2-3 months with participants being provided with interventions on a daily basis for 10-15 minutes each time. A possible outcome of this study is that your child may acquire many or all of his/her multiplication facts.

What is research?

- The goal of research is to learn new things in order to help people in the future. Investigators learn things by following the same plan with a number of participants, so they do not usually make changes to the plan for individual research participants. You, as an individual, may or may not be helped by volunteering for a research study.

Why am I being invited to take part in this research study?

We are asking you to take part in this research study because your child is a student in middle school at Andersen and is currently under performing in mathematics. In addition, your child has been identified as in need of math support.

What should I know about a research study?

- Someone will explain this research study to you.
- Whether or not you take part is up to you.
- You can choose not to take part.
- You can agree to take part and later change your mind.
- Your decision will not be held against you.
- You can ask all the questions you want before you decide.

Why is this research being done?

The purpose of this research is to examine the effects of a math intervention for improving multiplication fact fluency for middle school students using brief experimental analysis (BEA). BEA is the process of using single subject experimental design to briefly assess multiple academic interventions for individual students and utilizing visual analysis to determine any functional relation between treatment and the participant. This study will utilize BEA to answer the following three questions: (a) what is the effect of a math intervention identified during a brief experimental analysis on multiplication fact fluency for a middle school student, (b) what is the effect of the BEA identified intervention on maintenance and generalization to inverse multiplication facts and word problems, and (c) to what extent does the effect of the BEA identified intervention maintain overtime? A possible benefit for participating in this study is that your child may learn his/her multiplication facts.

How long will the research last?

We expect that your child will be in this research study till the end of the second quarter or until sufficient data is collected to determine an intervention effect but no later than the end of the school year. More specifically, your child's participation, if selected, will consist of daily intervention sessions ranging from 10-15 minutes each session with no more than 2 sessions per day. These sessions will occur within the school building (e.g. library, hallway, classroom, resource room, office).

Is there any way that being in this study could be bad for me?

We do not anticipate any risk to any student who participates in the project.

Will being in this study help me in any way?

We cannot promise any benefits to you or others from your taking part in this research. However, possible benefits include your child learning some or all of his/her multiplication facts.

What happens if I do not want to be in this research?

There are no known alternatives, other than deciding not to participate in this research study.

Detailed Information About This Research Study

The following is more detailed information about this study in addition to the information listed above.

How many people will be studied?
We expect to recruit 6 students maximum as participants in this study.

What happens if I say "*Yes, I want to be in this research*"?
If you agree to have your child participate in this study, he/she will need to complete a student assent form in order to become eligible participants. Thereafter, your child will be screened to meet requirements for this study (i.e. scoring 30 or less multiplication facts per minute). If they meet this screening, they will become study participants. Study participants' will be provided with math interventions by study personnel as described in the key information section above.

What happens if I say "Yes", but I change my mind later?
You can leave the research study at any time and no one will be upset by your decision. Choosing not to be in this study or to stop being in this study will not result in any penalty to you or loss of benefit to which you are entitled. This means that your choice not to be in this study will not negatively affect your child's right to continue receiving educational benefits.

Will it cost me anything to participate in this research study?
Taking part in this research study will not lead to any costs to you.

What happens to the information collected for the research?
Efforts will be made to limit the use and disclosure of your personal information, including research study and medical records, to people who have a need to review this information. We cannot promise complete confidentiality. Organizations that may inspect and copy your information include the Institutional Review Board (IRB), the committee that provides ethical and regulatory oversight of research, and other representatives of this institution, including those that have responsibilities for monitoring or ensuring compliance. Identifiers (e.g. numbers) will be assigned to each participant as a mean to provide additional layer of security and protection of sensitive information. Furthermore, information collected for this study will be kept in a locked and secured environment within the Educational Psychology department at the University of Minnesota, Twin Cities campus. Only identified personnel stated in this study will have access to these information.

Whom do I contact if I have questions, concerns or feedback about my experience?
This research has been reviewed and approved by an IRB within the Human Research Protections Program (HRPP). To share feedback privately with the HRPP about your research experience, call the Research Participants' Advocate Line at 612-625-1650 or go to https://research.umn.edu/units/hrpp/research-participants/questions-concerns. You are encouraged to contact the HRPP if:

- Your questions, concerns, or complaints are not being answered by the research team.

- You cannot reach the research team.
- You want to talk to someone besides the research team.
- You have questions about your rights as a research participant.
- You want to get information or provide input about this research.

Will I have a chance to provide feedback after the study is over?
The HRPP may ask you to complete a survey that asks about your experience as a research participant. You do not have to complete the survey if you do not want to. If you do choose to complete the survey, your responses will be anonymous.
If you are not asked to complete a survey, but you would like to share feedback, please contact the study team or the HRPP. See the "Investigator Contact Information" of this form for study team contact information and "Whom do I contact if I have questions, concerns or feedback about my experience?" of this form for HRPP contact information.
Can I be removed from the research?
The person in charge of the research study or the sponsor can remove you from the research study without your approval. Possible reasons for removal include your child rate of absence, moving or transferring to another school, or at the participants' request.

Signature Block for Capable Adult:

Your signature documents your permission to take part in this research. You will be provided a copy of this signed document.

_______________________________________ _______________
Signature of Participant Date

____
Printed Name of Participant

_______________________________________ _______________
Signature of Person Obtaining Consent Date

____
Printed Name of Person Obtaining Consent

Appendix B

Student Assent Form

Title of Research Study: ***The Effects of a Math Intervention Identified Using Brief Experimental Analysis for Middle School Students Struggling With Multiplication Fact Fluency***

Researcher: Jennifer J. McComas

Student Investigator: John Mouanoutoua

What is research?

Doctors and researchers are committed to your care and safety. There are important differences between research and treatment plans:

- The goal of research is to learn new things in order to help groups of kids in the future. Researchers learn things by asking a question, making a plan, and testing it.

Why am I being asked to take part in this research study?

A research study is usually done to find a better way to treat people or to understand how things work. You are being asked to take part in this research study because you are a middle school student at Andersen and have been identified as needing math support.

What should I know about being in a research study?

You do not have to be in this study if you do not want to do so. It is up to you if you want to participate and if you want to, talk to your parents about any questions or concerns you have about the study. You can choose not to take part now and change your mind later if you want. If you decide you do not want to be in this study, no one will be mad at you.

Why is this research being done?

In this study, I want to find out more about how brief experimental analysis can be use to identify math interventions that are effective for increasing multiplication facts for middle school students.

How long will the research last?

This research study is expected to last till the end of the second quarter or until sufficient data has been collected to determine an intervention effect.

What happens if I say "Yes, I want to be in this research"?

If it is okay with you and you agree to join this study, you will be asked to take a 1-minute multiplication facts screener to confirm your eligibility in this study. If you meet the criteria in this screener, you will become a participant.

What happens to the information collected for the research?

All information being collected will be confidential. In certain instances, researchers will share your information, including research study records, to only people who have a need to review this information. For example, sometimes researchers need to share information with the University or other people that work in research to make sure the researchers are following the rules.

Who can I talk to?

For questions about research appointments, the research study, research results, or other concerns, call the study team at:

Researcher Name: Jennifer J. McComas Researcher Affiliation: University of Minnesota, Twin Cities Email Address: jmmcomas@umn.edu	Study Staff (if applicable): John Mouanoutoua Email Address: mouan003@umn.edu

This research has been reviewed and approved by an Institutional Review Board (IRB), a group of people that look at the research before it starts. This group is part of the Human Research Protection Program (HRPP). To share concerns privately with the HRPP about your research experience, call the Research Participants' Advocate Line at 612-625-1650 or go to https://research.umn.edu/units/hrpp/research-participants/questions-concerns. You are encouraged to contact the HRPP if:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.
- You want to talk to someone besides the research team or your parents.
- You have questions about your rights as a research participant.
- You want to get information or provide feedback about this research.

Signature Block for Child Assent

Signature of child	Date
Printed name of child	
Printed name of person obtaining assent	Date
Signature of person obtaining assent	

Appendix C

Institutional Review Board Approval

UNIVERSITY OF MINNESOTA

Twin Cities Campus

Human Research Protection Program
Office of the Vice President for Research

D528 Mayo Memorial Building
420 Delaware Street S.E.
MMC 820
Minneapolis, MN 55455

Phone: 612-626-5654
Fax: 612-626-6061
Email: irb@umn.edu
http://www.research.umn.edu/subjects/

APPROVAL OF NEW STUDY

November 16, 2018

Jennifer McComas

jmccomas@umn.edu

Dear Jennifer McComas:

On 10/16/2018, the IRB reviewed the following submission:

Type of Review:	Initial Study
Title of Study:	The Effects of a Math Intervention Identified Using Brief Experimental Analysis for Students Struggling With Multiplication Fact Fluency
Investigator:	Jennifer McComas
IRB ID:	STUDY00003146
Sponsored Funding:	None
Grant ID/Con Number:	None
Internal UMN Funding:	None
Fund Management Outside University:	None
IND, IDE, or HDE:	None
Documents Reviewed with this Submission:	• parent consent form long version.pdf, Category: Consent Form; • certificate translation spanish cristina.JPG, Category: Other; • Parent consent form long version_Spanish.pdf, Category: Consent Form; • IRB protocol BEA for math intervention 11 12 18.docx, Category: IRB Protocol; • parent consent somali.pdf, Category: Consent Form; • intervention flyer, Category: Recruitment Materials; • MPLS REAA approval letter.pdf, Category: Letters of Support / Approvals (Location);

Driven to Discover℠

	• student assent form .pdf, Category: Consent Form; • student assent somali.pdf, Category: Consent Form; • certificate translation somali kamaal.JPG, Category: Other; • principal support letter fall study.pdf, Category: Letters of Support / Approvals (Location); • SCREENER.pdf, Category: Other; • sample survey.pdf, Category: Other; • Student assent form Spanish.pdf, Category: Consent Form;

The IRB determined that the criteria for approval have been met and that this study involves no greater than minimal risk

This study was approved under Expedited Category:

• (7) Research on individual or group characteristics or behavior or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

The IRB also made the following determinations for this study: This research has been approved to include children (individuals under 18 years of age) under 45 CFR 46.404 because the IRB finds that no greater than minimal risk to children is presented and that adequate provisions are made for soliciting the assent of the children and the permission of their parents or guardians.

The IRB approved the study from 11/16/2018 to 10/15/2019 inclusive. You will be sent a reminder from ETHOS to submit a Continuing Review submission for this study. You must submit your Continuing Review no later than 30 days prior to the last day of approval in order for your study to be reviewed and approved for another Continuing Review period. If Continuing Review approval is not granted before 10/15/2019, approval of this protocol expires immediately after that date.

You must also submit a Modification in ETHOS for review and approval prior to making any changes to this study.

If consent forms or recruitment materials were approved, those are located under the Final column in the Documents tab in the ETHOS study workspace.

In conducting this study, you are required to follow the requirements listed in the Investigator Manual (HRP-103), which can be found by navigating to the HRPP Toolkit Library on the IRB website.

For grant certification purposes, you will need the approval and last day of approval dates listed above and the Assurance of Compliance number which is FWA00000312 (Fairview Health Systems Research FWA00000325, Gillette Children's Specialty Healthcare FWA00004003).

Sincerely,

Bri Warner
IRB Anayst

We value feedback from the research community and would like to hear about your experience. The link below will take you to a brief survey that will take a minute or two to complete. The questions are basic, but your responses will help us better understand what we are doing well and areas that may require improvement. Thank you in advance for completing the survey.

Even if you have provided feedback in the past, we want and welcome your evaluation.

http://z.umn.edu/irbsurvey

Appendix D

Intervention Survey

Name: ____ Date: ______________________

Intervention: ____

Directions: Please answer the following questions by circling the appropriate rating scale from 1 (strongly disagree) to 6 (strongly agree).

1. This is a helpful way to teach math.

   1 2 3 4 5 6

2. This way to teach math is too hard for me.

   1 2 3 4 5 6

3. This way to teach math may be hard for other students.

   1 2 3 4 5 6

4. There are other ways to teach math to students that is better than this way.

   1 2 3 4 5 6

5. This way of teaching math is good to use with other students.

   1 2 3 4 5 6

6. I like this way of teaching math.

   1 2 3 4 5 6

7. I think teaching math this way will help other students learn their math.

   1 2 3 4 5 6

Appendix E

Screener

SCREENER

PLEASE PRINT YOUR NAME: ___________________________ DATE:__________

3 x 2	9 x 6	4 x 4	2 x 5	8 x 3
7 x 1	3 x 5	6 x 5	4 x 3	9 x 7
0 x 3	8 x 5	9 x 3	6 x 3	7 x 4
1 x 2	9 x 5	6 x 2	0 x 5	1 x 3
3 x 3	9 x 1	8 x 7	9 x 4	7 x 2
3 x 6	2 x 9	7 x 3	8 x 2	7 x 5
7 x 7	5 x 1	0 x 7	6 x 4	8 x 2
7 x 6	2 x 4	1 x 0	2 x 2	5 x 5

TOTAL # OF PROBLEMS CORRECT: _______ / TOTAL # OF PROBLEMS COMPLETED: _______

Appendix F

Incremental Rehearsal Protocol

Instructions: Check the corresponding box for each observed intervention procedural step then sign, date, and compute below.

Materials: multiplication flash cards, designated worksheet (IFA), pencil, timer
IR Intervention Preparations:

a. Choose 9 **known** facts from participant's fact assessment (see student chart)
b. Choose 5 **unknown** facts from participant's fact assessment (see student chart)
c. Place **known** and **unknown** facts in separate piles facing down
   (known facts = pile B; unknown facts = pile A)

<u>Intervention Procedural Steps</u>

1. Set the timer for 5 minutes.
2. Show the participant an unknown fact from pile "A", state the problem with the answer to the participant, and have the student say the problem and answer. ☐
3. Place this 1st unknown fact face down (starting a new pile "C") and present a known fact from pile "B" to the participant asking him/her to just state the answer. Once the answer is given, place this card on top of pile "C". ☐
4. Pick up pile "C" and present all cards one at a time (in the order it was placed; the unknown fact should always be the first card) to the participant ensuring that he/she states the problem with the answer of the corresponding unknown fact and only state the answer to known facts. ☐
5. Repeat steps 3 and 4 until all the cards from pile "B" have been used. ☐
6. Discard the last known fact that was introduced and select a new unknown from pile "B" (this will become the 2nd unknown fact) and have the participant repeat the problem and answer after you. ☐
7. Repeat steps 3-7 until all cards in pile "A" is used at least once or until the allotted 5 minutes have ended. ☐
8. At the end of the session give the student a 1-minute posttest faced down and say "***You have 1 minute to complete this test. Complete the problems from left to right without skipping You will not be penalized for getting each problem wrong. When the time is up, I will tell you to stop. Do your best, ready begin***". Start the 1-minute timer. ☐
11. When the timer goes off, say "***Stop, put your pencil down***" and collect the posttest. ☐

Totaled # of boxes checked: ____________

Signature: ____ Date: ____________________

Appendix G

Math to Mastery Protocol

Instructions: Check the corresponding box for each observed intervention procedural step then sign, date, and compute below.

Materials: timer, 2 copies of intervention worksheets, graphing papers, student progress monitoring chart

<u>Intervention Procedural Steps</u>

1. Set the timer to 5 minutes and take out a pair of worksheet and graphing papers; one for you and one for the student and say,

   "I am going to complete the first few facts using a concrete representational abstract (CRA) method and a few multiplication strategies. Please follow along with me on your paper while I complete them. When I'm done, you will complete some facts on your own for each row. " ☐
2. Model how to compute the first problem then allow the participant to complete the row of facts independently. ☐
3. Provide immediate corrective feedback if necessary while the student completes his/her row. After the student completes his/her row, give the student their self-monitoring chart and say,

   "Let's check and count up the problems you got correct and record it in your chart." ☐
4. After the student has recorded their results, say

   " Great job, let's continue to do the next row. " ☐
5. Repeat step 3-4 until the 5-minute intervention time has ended or after completing all the problems on the instructional worksheet. ☐
6. Collect the instructional worksheet and give the participant an intervention fact assessment and say:

   "You will have one minute to complete as many problems as you can on this worksheet. Complete the problems from left to right without skipping around. You will not be penalized for getting the answers wrong. I cannot help you, but I will let you know when the time is up and you need to put your pencil down. You may begin." Begin the 1-minute timer. ☐
7. When the timer is up, say ***"Stop, put your pencil down"*** and collect the assessment. ☐

Totaled # of boxes checked: ____________

Signature: ____ Date: _________________

Appendix H

Reward Protocol

Instructions: Check the corresponding box for each observed intervention procedural step then sign, date, and compute below.

Materials: pencils, timer, (2) randomly selected Reward worksheets (label them below), bag of goodies

Reward Intervention Preparations

a. Worksheet ___________ b. Worksheet ___________

Intervention Procedural Steps

1. Set a timer for 5 minutes. ☐
2. Give a worksheet to the participant face down and say "***You have 1 minute to complete as many problems as possible. Complete the problems from left to right without skipping. You will not be penalized for getting any problems wrong. When the time is up, I will tell you to stop. Do your best, ready begin***".
   Start the 1-minute timer. ☐
3. When the timer goes off, say "***Stop, put your pencil down***" and collect the worksheet. ☐
4. Calculate the total number of problems correct per minute (PCPM) and add 30%. Write the goal here: _______ ☐
5. Show the goal to the participant and say ***"If you can get this number of PCPM on this next trial then you can get a reward".*** ☐
6. Have the student pick a reward from the bag of goodies and set it aside. ☐
7. Give the second worksheet to the participant face down and say "***You have 1 minute to complete as many problems as possible. Complete the problems from left to right without skipping. You will not be penalized for getting any problems wrong. When the time is up, I will tell you to stop. Do your best, ready begin***".
   Start the 1-minute timer. ☐

8. When the timer goes off, say "***Stop, put your pencil down***" and collect the worksheet. ☐

9. Calculate the total number of problems correct per minute (PCPM) and if the participant met the goal then give him/her the reward, if not, give him/her a pencil or a an eraser from the bag of goodies. ☐

Totaled # of boxes checked: ____________

Signature: ____ Date: ______________

Appendix I

Baseline and Generalization Protocol

Instructions: Check the corresponding box for each observed intervention procedural step then sign, date, and compute below.

Materials: assessment worksheet, timer, pencils

Check one:

Baseline probe: _______
Inverse probe: _______
Division probe: _______

Procedural Steps

1. Give the participant the assessment worksheet and say:
   "You will have one minute to complete as many problems as you can on this worksheet. Complete the problems from left to right without skipping around. You will not be penalized for getting the answers wrong. I cannot help you, but I will let you know when the time is up and you need to put your pencil down. You may begin." Begin the 1-minute timer. ☐
2. When the timer is up, say ***"Stop, put your pencil down"*** and collect the assessment. ☐

Totaled # of boxes checked: _____________

Signature: ____ Date: _______________